MAT2440, Quiz4, Spring2025

ID:______________________________ Name:______________________________

1. Give a proof by **contraposition** of the theorem "If n^2 is an even integer, then n is even."

p q

To show "p implies q" by ¬q implies ¬p" (the contraposition),

we assume n is not even $\Rightarrow$ n is odd.
(¬q)

Since n is odd, we have $n = 2k+1$ (k is an integer).

Then $n^2 = (2k+1)^2 = 4k^2+4k+1 = 2(2k^2+2k)+1$

which is an odd integer. (¬p)

(We show "if n is odd, then n^2 is odd" which proves

"if n^2 is even, then n is even" is true)

2. Give a proof by **contradiction** of the theorem "If n^2 is an even integer, then n is even."

p q

To show "$p \to q$" is true by contradiction

We have p is true and assume ¬q is true which means

n^2 is even and n is odd

If n is odd, we let $n = 2k+1$ and we have

$n^2 = (2k+1) = 4k^2+4k+1 = 2(2k^2+2k)+1$

which is an odd integer.

However, based on the given condition, n^2 is even and

this makes "n^2 is odd" a contradiction

which came from the assumption n is odd

Therefore, this assumption "n is odd" is false and

n should be even.