

# MAT2440, Quiz3, Spring2025

ID: \_\_\_\_\_ Name: \_\_\_\_\_

1. Let  $C(x)$ ,  $D(x)$ , and  $F(x)$  be the statements ‘ $x$  has a cat,’ ‘ $x$  has a dog,’ and ‘ $x$  has a ferret,’ respectively, where the domain consists of all students in your class.

(a) Translate the statement into English: “ $\neg \exists x(C(x) \wedge D(x) \wedge F(x))$ ”

No one in your class has a cat, and a dog, and a ferret.

(b) Express the statements using quantifiers; logical connectives; and  $C(x)$ ,  $D(x)$ , and  $F(x)$ :

“Some student in your class has a cat, and a ferret, but not a dog.”

$\exists x (C(x) \wedge F(x) \wedge \neg D(x))$

2. Use rules of inference to show that the premises “If you send me an email, then I will finish writing the program,” “If you don’t send me an email, then I will go to sleep early,” “If I go to sleep early, then I will wake up feeling refreshed,” lead to the conclusion “If I don’t finish writing the program, then I will wake up feeling refreshed.”

(Please use the following propositions to show your answer: e: “you send me an email,” f: “I will finish writing the program,” s: “I will go to sleep early,” w: “I will wake up feeling refreshed”)

Sol

$e \rightarrow f$	(given) Premise #1	<p style="color: orange;">③ premise #5 is a contrapositive of #1.</p> <p style="color: green;">To get “<math>\neg f \rightarrow s</math>”, we need premise #5: <math>\frac{\neg f \rightarrow \neg e}{\neg e \rightarrow s} \therefore \neg f \rightarrow s</math></p> <p style="color: purple;">① To get this conclusion we need premise #4: since <math>\neg f \rightarrow s</math> <math>\frac{s \rightarrow w}{\therefore \neg f \rightarrow w}</math></p>
$\neg f \rightarrow \neg e$	(given) Premise #5	
$\neg e \rightarrow s$	(given) Premise #2	
$\neg f \rightarrow s$	(given) Premise #4	
$s \rightarrow w$	(given) Premise #3	
$\therefore \neg f \rightarrow w$		Conclusion