Mat1372 HW13

5.15 Identify hypotheses, Part I. Write the null and alternative hypotheses in words and then symbols for each of the following situations.

(a) A tutoring company would like to understand if most students tend to improve their grades (or not) after they use their services. They sample 200 of the students who used their service in the past year and ask them if their grades have improved or declined from the previous year.

(b) Employers at a firm are worried about the effect of March Madness, a basketball championship held each spring in the US, on employee productivity. They estimate that on a regular business day employees spend on average 15 minutes of company time checking personal email, making personal phone calls, etc. They also collect data on how much company time employees spend on such non-business activities during March Madness. They want to determine if these data provide convincing evidence that employee productivity changed during March Madness.

Sol (a) H_0: $p = 0.5$ (half of their grades improved and the half did not)

H_A: $p \neq 0.5$ (Either a majority or a minority of students' grades improved)

(b) H_0: $\mu = 15$ (the mean of time is exactly 15 mins)

H_A: $\mu \neq 15$ (the mean is more or less than 15 mins)

5.17 Online communication. A study suggests that 60% of college student spend 10 or more hours per week communicating with others online. You believe that this is incorrect and decide to collect your own sample for a hypothesis test. You randomly sample 160 students from your dorm and find that 70% spent 10 or more hours a week communicating with others online. A friend of yours, who offers to help you with the hypothesis test, comes up with the following set of hypotheses. Indicate any errors you see.

$$H_0 : \hat{p} < 0.6$$
$$H_A : \hat{p} > 0.7$$

Sol: ① The hypothesis should be about population proportion p, not sample proportion $\hat{p}$.

② The null hypothesis H_0 should be an "equal" statement, that is, H_0: $p = 0.6$

③ The alternative hypothesis H_A should be non-H_0, that is,

H_A: $p \neq 0.6$

5.19 Cyberbullying rates. Teens were surveyed about cyberbullying, and 54% to 64% reported experiencing cyberbullying (95% confidence interval).[23] Answer the following questions based on this interval.

(a) A newspaper claims that a majority of teens have experienced cyberbullying. Is this claim supported by the confidence interval? Explain your reasoning.

(b) A researcher conjectured that 70% of teens have experienced cyberbullying. Is this claim supported by the confidence interval? Explain your reasoning.

(c) Without actually calculating the interval, determine if the claim of the researcher from part (b) would be supported based on a 90% confidence interval?

Sol: 95% confidence interval: (54%, 64%)

(a) Since the entire interval lies above 50%, then this claim is reasonable

(b) The value of 70% is not in the interval, so this conjecture is wrong

(c) NO. since 90% confidence interval will be narrower than a 95% confidence interval, we can say that 70% will not be in 90% confidence interval.

5.21 Minimum wage, Part I. Do a majority of US adults believe raising the minimum wage will help the economy, or is there a majority who do not believe this? A Rasmussen Reports survey of a random sample of 1,000 US adults found that 42% believe it will help the economy.[24] Conduct an appropriate hypothesis test to help answer the research question.

Sol: Let's use the framework from the textbook:

Prepare Set up hypothesis

H_0: $P = 0.5$ (no majority)

H_A: $P \neq 0.5$ (either majority or minority)

We here use a significance level $\alpha = 0.05$.

Check Independent: Yes, since this survey is a random sample

success–failure condition: null value $p_0 = 0.5$, sample size $n = 1000$

$n p_0 = 1000 \cdot 0.5 = 500 \geq 10$

$n(1-p_0) = 1000 \cdot 0.5 = 500 \geq 10$ } pass the condition.

Calculate $SE = \sqrt{\frac{p(1-p)}{n}} = \sqrt{\frac{0.5(1-0.5)}{1000}} = 0.016$.

For $\hat{p} = 0.42$, we have $Z = \frac{0.42 - 0.5}{0.016} = -5$, then we have

the p-value $= 2 \times P(Z < -5) = 2 \times (0,0002) = 0,0004$

Conclude since p-value $= 0,0004 < 0,05 = \alpha$, then we reject H_0 and conclude that the people who believe wage raising is not 50%.

Furthermore, since the observed value is 42% which is less than 50%, so we think that the people who think this way is minority.

5.23 Working backwards, Part I. You are given the following hypotheses:

$$H_0 : p = 0.3$$
$$H_A : p \neq 0.3$$

We know the sample size is 90. For what sample proportion would the p-value be equal to 0.05? Assume that all conditions necessary for inference are satisfied.

Sol: $n = 90$. $p_0 = 0,3$. $SE = \sqrt{\frac{p(1-p)}{n}} \approx \sqrt{\frac{p_0(1-p_0)}{90}} = \sqrt{\frac{0,3 \cdot 0,7}{90}} = 0,048$.

Let $\hat{p}$ be the sample proportion such that

$Z_{\hat{p}} = \frac{\hat{p} - 0,3}{0,048}$ and $P(Z < Z_{\hat{p}}) = \frac{0,05}{2} = 0,025$

$\Rightarrow Z_{\hat{p}} = -1,96 \Rightarrow \frac{\hat{p} - 0,3}{0,048} = -1,96 \Rightarrow \hat{p} = 0,206$

or $P(Z > Z_{\hat{p}}) = 0,025 \Rightarrow Z_{\hat{p}} = 1,96 \Rightarrow \hat{p} = 0,394$