Mat 1372 HW7

3.31 Hearts win. In a new card game, you start with a well-shuffled full deck and draw 3 cards without replacement. If you draw 3 hearts, you win \$50. If you draw 3 black cards, you win \$25. For any other draws, you win nothing.

(a) Create a probability model for the amount you win at this game, and find the expected winnings. Also compute the standard deviation of this distribution.

(b) If the game costs \$5 to play, what would be the expected value and standard deviation of the net profit (or loss)? *(Hint: profit = winnings − cost; X − 5)*

(c) If the game costs \$5 to play, should you play this game? Explain.

Sol (a) Assume X is the random variable of the money you win.

① $P(X=50) = P(\text{draw 3 hearts w/o replacement}) = \frac{13}{52}\cdot\frac{12}{51}\cdot\frac{11}{50}$
$= 0.0129$

② $P(X=25) = P(\text{draw 3 black w/o replacement}) = \frac{26}{52}\cdot\frac{25}{51}\cdot\frac{24}{50} = 0.1176$

③ $P(X=0) = P(\text{other than ① \& ②}) = 1-(0.0129+0.1176) = 0.8695$

$E(X) = 50\cdot P(X=50) + 25\cdot P(X=25) + 0\cdot P(X=0)$
$= 50\cdot 0.0129 + 25\cdot 0.1176 + 0\cdot 0.8695 = \$3.59$

$Var(X) = (50-E(X))^2\cdot P(X=50) + (25-E(X))^2\cdot P(X=25) + (0-E(X))^2 P(X=0)$
$= 27.87 + 53.93 + 11.21 = 93.01$

$SD(X) = \sqrt{var(X)} = \sqrt{93.01} = 9.64$

(b) Let X be the money you win, then net profit can be denoted as $X-5$. We have

$E(X-5) = E(X) - 5 = 3.59 - 5 = -\$1.41$

$SD(X-5) = SD(X) = 9.64$

(c) No. since your expected profit is a negative number: -1.41. so on average you expect lose money.

3.32 Is it worth it? Andy is always looking for ways to make money fast. Lately, he has been trying to make money by gambling. Here is the game he is considering playing: The game costs \$2 to play. He draws a card from a deck. If he gets a number card (2-10), he wins nothing. For any face card (jack, queen or king), he wins \$3. For any ace, he wins \$5, and he wins an *extra* \$20 if he draws the ace of clubs.

(a) Create a probability model and find Andy's expected profit per game.

(b) Would you recommend this game to Andy as a good way to make money? Explain.

Sol (a) Assume X be the profit Tom makes:

pay \$2 to play this game

$P(X=0-2)=P(X=-2)=P(\text{draw a 2-10 card})=\frac{9\cdot 4}{52}=\frac{36}{52}$

$P(X=3-2)=P(X=1)=P(\text{face card})=\frac{4\cdot 3}{52}=\frac{12}{52}$

$P(X=5-2)=P(X=3)=P(\text{Ace of spade, heart, diamond})=\frac{3}{52}$

$P(X=25-2)=P(X=23)=P(\text{Ace of clubs})=\frac{1}{52}$

$\$5$ + extra $\$20$

$E(X)=(-2)P(X=-2)+1\cdot P(X=1)+3\cdot P(X=3)+23\cdot P(X=23)$

$=(-2)\frac{36}{52}+1\cdot\frac{12}{52}+3\cdot\frac{3}{52}+23\cdot\frac{1}{52}=-0.54$

(b) NO. he is expected to lose money on average.

3.33 Portfolio return. A portfolio's value increases by 18% during a financial boom and by 9% during normal times. It decreases by 12% during a recession. What is the expected return on this portfolio if each scenario is equally likely?

Assume X is the porfolio's value.

"Each scenario is equally likely" implies

$P(X=0.18)=\frac{1}{3}$, $P(X=0.09)=\frac{1}{3}$, $P(X=-0.12)=\frac{1}{3}$

Boom normal recession.

$E(X)=0.18\cdot P(X=0.18)+0.09\cdot P(X=0.09)+(-0.12)\cdot P(X=-0.12)$

$=0.18\cdot\frac{1}{3}+0.09\cdot\frac{1}{3}+(-0.12)\frac{1}{3}=0.05$

The expected return is a 5% increase in value

3.34 Baggage fees. An airline charges the following baggage fees: \$25 for the first bag and \$35 for the second. Suppose 54% of passengers have no checked luggage, 34% have one piece of checked luggage and 12% have two pieces. We suppose a negligible portion of people check more than two bags.

(a) Build a probability model, compute the average revenue per passenger, and compute the corresponding standard deviation.

(b) About how much revenue should the airline expect for a flight of 120 passengers? With what standard deviation? Note any assumptions you make and if you think they are justified.

Sol: Assume X be the random variable of the fees.

(a) $P(X=0) = P(\text{ppl without bag}) = 0.54$

$P(X=25) = P(\text{ppl with 1 bag}) = 0.34$

$P(X=35+25) = P(X=60) = P(\text{ppl with 2 bags}) = 0.12$

$$\text{Average} = E(X) = 0\cdot P(X=0) + 25\cdot P(X=25) + 60\cdot P(X=60)$$
$$= 0\cdot 0.54 + 25\cdot 0.34 + 60\cdot 0.12 = \$15.70$$

$$Var(X) = (0-E(X))^2\cdot P(X=0) + (25-E(X))^2\cdot P(X=25) + (60-E(X))^2 P(X=60)$$
$$= 246.49\cdot 0.54 + 86.49\cdot 0.34 + 1962.49\cdot 0.12$$
$$= \$398.01$$

$$SD(X) = \sqrt{Var(X)} = \sqrt{398.01} = \$19.95$$

(b) Assum all fliers are independent. Thus, for all fliers, their expected fees are the same: $E(X) = 15.70$.

There are 120 fliers, and their random variable of fees are $X_1, X_2, X_3, \dots, X_{120}$, respectively.

Then $E(X_1 + X_2 + \dots + X_{120}) = E(X_1) + E(X_2) + \dots + E(X_{120})$

$$= 120\cdot 15.7 = \$1884.$$

$$Var(X_1 + X_2 + \dots + X_{120}) = Var(X_1) + Var(X_2) + \dots + Var(X_{120})$$
$$= 120\cdot 398.01 = \$47761.20$$

$$SD(X_1 + \dots + X_2) = \sqrt{Var(X_1 + X_2 + \dots + X_{120})} = \sqrt{47761.20}$$
$$= \$218.54$$

3.36 European roulette. The game of European roulette involves spinning a wheel with 37 slots: 18 red, 18 black, and 1 green. A ball is spun onto the wheel and will eventually land in a slot, where each slot has an equal chance of capturing the ball. Gamblers can place bets on red or black. If the ball lands on their color, they double their money. If it lands on another color, they lose their money.

(a) Suppose you play roulette and bet \$3 on a single round. What is the expected value and standard deviation of your total winnings?

(b) Suppose you bet \$1 in three different rounds. What is the expected value and standard deviation of your total winnings?

(c) How do your answers to parts (a) and (b) compare? What does this say about the riskiness of the two games?

Sol: (a) Assume X be the money winning in a single round.

$$P(X=0-3)=P(X=-3)=P(\text{bets on wrong color})=\frac{19}{37}$$

$$P(X=2\cdot3-3)=P(X=3)=P(\text{bets on the right color})=\frac{18}{37}$$

$$E(X)=(-3)P(X=-3)+3\cdot P(X=3)$$

$$=(-3)\cdot\frac{19}{37}+3\cdot\frac{18}{37}=-0.081$$

$$Var(X)=(-3-E(X))^2\frac{19}{37}+(3-E(X))^2\cdot\frac{18}{37}$$

$$=9.4926\cdot\frac{19}{37}+8.5206\cdot\frac{18}{37}=9$$

$$SD(X)=\sqrt{Var(X)}=\sqrt{9}=3$$

(b) Let Y be the money winning for betting \$1 in a single round

$$P(Y=0-1)=P(Y=-1)=P(\text{bets on the wrong color})$$

$$P(Y=2\cdot1-1)=P(Y=1)=P(\text{bets on the right color})$$

Then $Y=\frac{1}{3}X$

$$\Rightarrow E(Y)=E\left(\frac{1}{3}X\right)=\frac{1}{3}E(X)=\frac{-0.081}{3} \text{ and}$$

$$Var(Y)=Var\left(\frac{1}{3}X\right)=\frac{1}{9}Var(X)=\frac{9}{9}=1$$

Thus, if you play three times, then you get

$$E(Y)+E(Y)+E(Y)=3\cdot\frac{-0.081}{3}=-0.081$$

$$V(Y)+V(Y)+V(Y)=3\cdot1=3$$

$$SD(Y+Y+Y)=\sqrt{V(Y+Y+Y)}=\sqrt{3}=1.73$$

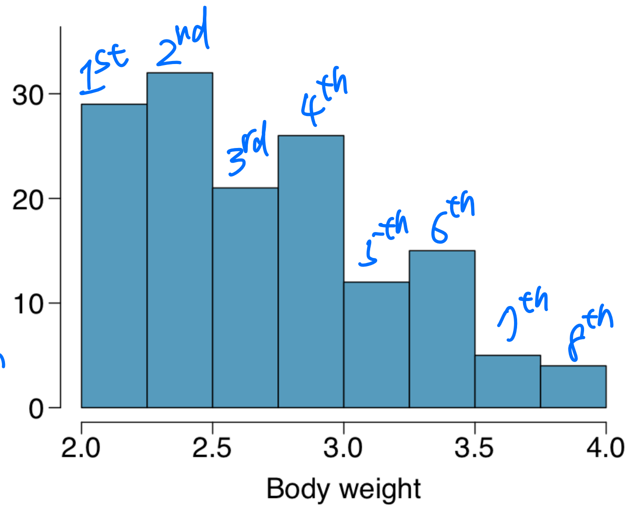
(c) Betting \$3 one time and \$1 three times have the expected money winning, but \$3 one time is risker than \$1 three time because the SD(X) is larger than SD(Y+Y+Y)

3.37 Cat weights. The histogram shown below represents the weights (in kg) of 47 female and 97 male cats.[62]

(a) What fraction of these cats weigh less than 2.5 kg?

(b) What fraction of these cats weigh between 2.5 and 2.75 kg?

(c) What fraction of these cats weigh between 2.75 and 3.5 kg?



Sol:

(a) $P(\text{Weight} < 2.5) = \dfrac{1^{st}\text{ bar} + 2^{nd}\text{ bar}}{\text{total cats}}$

$= \dfrac{29+32}{47+97} = 0.4236$

(b) $P(2.5 < \text{weight} < 2.75) = \dfrac{3^{rd}\text{ bar}}{144} = \dfrac{21}{144} = 0.1458$

(c) $P(2.75 < W < 3.5) = \dfrac{4^{th}\text{ bar} + 5^{th}\text{ bar} + 6^{th}\text{ bar}}{144} = \dfrac{26+12+15}{144} = 0.3681$