Mat 1372 HW6

3.24 Socks in a drawer. In your sock drawer you have 4 blue, 5 gray, and 3 black socks. Half asleep one morning you grab 2 socks at random and put them on. Find the probability you end up wearing

(a) 2 blue socks
(b) no gray socks
(c) at least 1 black sock
(d) a green sock
(e) matching socks

(a) Total $4+5+3=12$ socks. $P(2 \text{ blue}) = \frac{4}{12} \cdot \frac{3}{11} = 0.0909$

(b) $P(\text{no gray}) = P(\text{1st not gray}) \cdot P(\text{2nd not gray} \mid \text{1st not gray})$
$= \frac{7}{12} \cdot \frac{6}{11} = 0.3182$

(c) $P(\text{at leat 1 black}) = 1 - P(\text{no black}) = 1 - \frac{9}{12} \cdot \frac{8}{11} = 1 - 0.5455$
$= 0.4545$

(d) $P(\text{a green}) = 0$, there is no green sock in the draw

(e) $P(\text{matching socks}) = P(2 \text{ blue}) + P(2 \text{ grey}) + P(2 \text{ black})$
$= \frac{4}{12} \cdot \frac{3}{11} + \frac{5}{12} \cdot \frac{4}{11} + \frac{3}{12} \cdot \frac{2}{11} = \frac{38}{132}$
$= 0.2879$

3.26 Books on a bookshelf. The table below shows the distribution of books on a bookcase based on whether they are nonfiction or fiction and hardcover or paperback.

		Format		
		Hardcover	Paperback	Total
Type	Fiction	13	59	72
	Nonfiction	15	8	23
	Total	28	67	95

(a) Find the probability of drawing a hardcover book first then a paperback fiction book second when drawing without replacement.

(b) Determine the probability of drawing a fiction book first and then a hardcover book second, when drawing without replacement.

(c) Calculate the probability of the scenario in part (b), except this time complete the calculations under the scenario where the first book is placed back on the bookcase before randomly drawing the second book.

(d) The final answers to parts (b) and (c) are very similar. Explain why this is the case.

(a) $P(\text{hard cover first, paper fiction second})$
$= P(\text{hard cover 1st}) \cdot P(\text{paper fiction 2nd} \mid \text{hardcover first})$
$= \frac{28}{95} \cdot \frac{59}{94} = 0.1850$

without replacement $95-1=94$ books in the 2nd draw

(b) P(fiction first, hardcover second)

= P(fiction hardcover 1st, hardcover second) + P(fiction paper 1st, hardcover 2nd)

$= \frac{13}{95} \cdot \frac{27}{94} + \frac{59}{95} \cdot \frac{28}{94} = 0{,}2243$

(c) Draw with replacement.

P(fiction hardcover 1st, hardcover 2nd) + P(fiction paper 1st, hardcover 2nd)

$= \frac{13}{95} \cdot \frac{28}{95} + \frac{59}{95} \cdot \frac{28}{95} = 0{,}2234$

(d) When drawing rate is less than 10% (2 out of 95 books), the probability with / without replacement will be very similar.

3.28 The birthday problem. Suppose we pick three people at random. For each of the following questions, ignore the special case where someone might be born on February 29th, and assume that births are evenly distributed throughout the year.

(a) What is the probability that the first two people share a birthday?

(b) What is the probability that at least two people share a birthday?

p(first) = 1

(a) P (first 2 people share a bday) $= \frac{1}{365}$ (first one decides which date, and the second picked the same date)

(b) P(2 people share the same bday)

$= 1 - P$ (no one shares the same bday) $= 1 - \frac{365}{365} \cdot \frac{364}{365} \cdot \frac{363}{365} = 0{,}0082$

(just like a "with replacement" drawing)

4.4.3 Homework

1. You are going to a benefit dinner, and need to decide before the dinner what you want for salad, main dish, and dessert. You have 2 different salads to choose from, 3 main dishes, and 5 desserts. How many different meals are available?

$2 \times 3 \times 5 = 30$ combinations

salad, main dish, dessert

2. How many different phone numbers are possible in the area code 928?

There are 10 digits for a phone numbers. If area code is fixed, we have 7 digits left to be filled in:

928 _ _ _ _ _ _ _

and for each digit, there are 10 options (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

Thus, the combinations are $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = 10^7$

3. You are opening a T-shirt store. You can have long sleeves or short sleeves, three different colors, five different designs, and four different sizes. How many different shirts can you make?

$$\underset{\text{long/short sleeves}}{2} \times \underset{\text{color}}{3} \times \underset{\text{design}}{5} \times \underset{\text{size}}{4} = 120$$

4. The California license plate has one number followed by three letters followed by three numbers. How many different license plates are there?

_   _ _ _   _ _ _

number   letters   numbers

$10 \cdot 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10 = 26^3 \cdot 10^4$.

9. You have a group of twelve people. You need to pick a president, treasurer, and secretary from the twelve. How many different ways can you do this?

Assume these three roles cannot be the same person.

$12 \times 11 \times 10 = 1320.$

president

treasurer (12−1, 1 person being president)

secretary (12−2, 1 being president and 1 being treasurer)

10. A baseball team has a 25-person roster. A batting order has nine people. How many different batting orders are there?

Since the order of 9 people doesn't matter, then we use combination formular $_{25}C_9$ or $\binom{25}{9} = \frac{25!}{9!(25-9)!} = \frac{25!}{9!16!}$

11. An urn contains five red balls, seven yellow balls, and eight white balls. How many different ways can you pick two red balls?

Assume we at least pick 2 balls. without replacement and stop when we have 2 red balls,

① $r, r = {}_5C_2$ → 7 yellow + 8 white = 15

② $r, \_, r = {}_5C_2 \cdot {}_{15}C_1$

③ $r, \_, \_, r = {}_5C_2 \cdot {}_{15}C_2$

④ $r, \_, \_, \_, r = {}_5C_2 \cdot {}_{15}C_3$

⑤ $r, \_, \_, \_, \_, r = {}_5C_2 \; {}_{15}C_4$

⋮

⑯ $r, \_, \_, \_, \_, \_, \_, \_, \_, \_, \_, \_, \_, \_, \_, \_, r$
$= {}_5C_2 \; {}_{15}C_{15}$

Then we sum them up and get

$${}_5C_2\left(1 + {}_{15}C_1 + {}_{15}C_2 + {}_{15}C_3 + {}_{15}C_4 + \cdots + {}_{15}C_{13} + {}_{15}C_{14} + {}_{15}C_{15}\right)$$

12. How many ways can you choose seven people from a group of twenty?

Since the order of picking 7 people doesn't matter, then we have

$${}_{12}C_7 = \frac{12!}{7!\,5!}$$