Mat 1372 HW5

**3.13 Joint and conditional probabilities.** $P(A) = 0.3$, $P(B) = 0.7$

(a) Can you compute P(A and B) if you only know P(A) and P(B)?

(b) Assuming that events A and B arise from independent random processes,

i. what is P(A and B)?

ii. what is P(A or B)?

iii. what is P(A|B)?

(c) If we are given that P(A and B) = 0.1, are the random variables giving rise to events A and B independent?

(d) If we are given that P(A and B) = 0.1, what is P(A|B)?

(a) NO, We don't know if A and B were independent or not.

(b) Since A, B are independent, then

i) $P(A \text{ and } B) = P(A) \cdot P(B) = 0.3 \cdot 0.7 = 0.21$

ii) $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$= 0.7 + 0.3 - 0.21 = 0.79$

iii) $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)} = \dfrac{0.21}{0.3} = 0.7$

(c) NO, if $P(A \text{ and } B) = 0.1 \neq P(A) \cdot P(B)$, then A and B are not independent.

(d) If $P(A \text{ and } B) = 0.1$, then $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)} = \dfrac{0.1}{0.7} = 0.143$

**3.16 Health coverage, relative frequencies.** The Behavioral Risk Factor Surveillance System (BRFSS) is an annual telephone survey designed to identify risk factors in the adult population and report emerging health trends. The following table displays the distribution of health status of respondents to this survey (excellent, very good, good, fair, poor) and whether or not they have health insurance.

| | | *Health Status* | | | | | |
|---|---|---|---|---|---|---|---|
| | | Excellent | Very good | Good | Fair | Poor | Total |
| *Health* | No | 0.0230 | 0.0364 | 0.0427 | 0.0192 | 0.0050 | 0.1262 |
| *Coverage* | Yes | 0.2099 | 0.3123 | 0.2410 | 0.0817 | 0.0289 | 0.8738 |
| | Total | 0.2329 | 0.3486 | 0.2838 | 0.1009 | 0.0338 | 1.0000 |

(a) Are being in excellent health and having health coverage mutually exclusive?

(b) What is the probability that a randomly chosen individual has excellent health?

(c) What is the probability that a randomly chosen individual has excellent health given that he has health coverage?

(d) What is the probability that a randomly chosen individual has excellent health given that he doesn't have health coverage?

(e) Do having excellent health and having health coverage appear to be independent?

(a) NO, there are 20.99% of people in excellent health and having health cov.

(b) $P(\text{excellent health}) = 0.2329$

(c) $P(\text{excellent health} \mid \text{having health coverage}) = \dfrac{0.2099}{0.8738} = 0.24$

(d) P(excellent health | without health coverage) $= \frac{0.0230}{0.1262} = 0.18$

(e) P(excellent health and having coverage) $= 0.2099$

P(excellent health) $\times$ P(having coverage) $= 0.2329 \times 0.8738 = 0.2035$

Since P(excellent health and having coverage) $\neq$ P(Ex. health) $\cdot$ P(coverage),

then they are NOT independent.

**3.18 Assortative mating.** Assortative mating is a nonrandom mating pattern where individuals with similar genotypes and/or phenotypes mate with one another more frequently than what would be expected under a random mating pattern. Researchers studying this topic collected data on eye colors of 204 Scandinavian men and their female partners. The table below summarizes the results.[42]

| | | *Partner (female)* | | | |
|---|---|---|---|---|---|
| | | Blue | Brown | Green | Total |
| *Self (male)* | Blue | 78 | 23 | 13 | 114 |
| | Brown | 19 | 23 | 12 | 54 |
| | Green | 11 | 9 | 16 | 36 |
| | Total | 108 | 55 | 41 | 204 |

(a) What is the probability that a randomly chosen male respondent or his partner has blue eyes?

(b) What is the probability that a randomly chosen male respondent with blue eyes has a partner with blue eyes?

(c) What is the probability that a randomly chosen male respondent with brown eyes has a partner with blue eyes? What about the probability of a randomly chosen male respondent with green eyes having a partner with blue eyes?

(d) Does it appear that the eye colors of male respondents and their partners are independent? Explain your reasoning.

(a) P(male or his partner has blue eyes)

$= \frac{(\text{\# of male with blue}) + (\text{\# of his partner with blue}) - (\text{both blue})}{\text{total number}}$

$= \frac{114 + 108 - 78}{204} = 0.7059$

(b) P(a partner with blue eyes | male has blue eyes) $= \frac{78}{114} = 0.6842$

(c) P(a partner with blue eyes | male with brown) $= \frac{19}{54} = 0.3519$

P(a partner with blue | male with green) $= \frac{11}{36} = 0.3056$

(d) P(male blue and his partner blue) $= \frac{78}{204}$

P(male blue) $= \frac{114}{204}$ . P(partner blue) $= \frac{108}{204}$

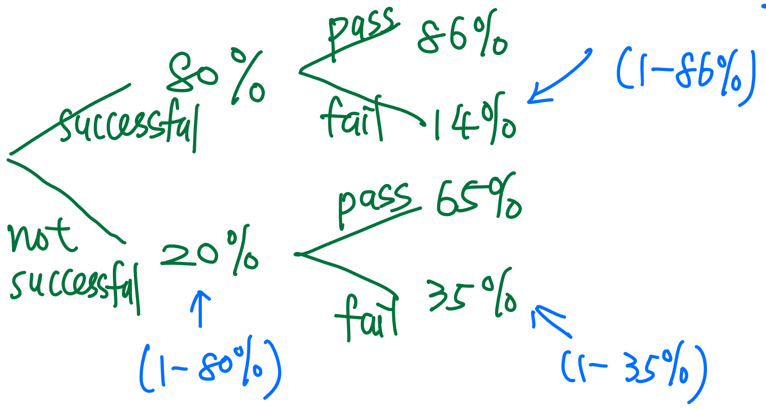
Since P(male blue and partner blue) $\neq$ P(male blue) $\cdot$ P(partner blue), then they are NOT independent.

**3.19 Drawing box plots.** After an introductory statistics course, 80% of students can successfully construct box plots. Of those who can construct box plots, 86% passed, while only 65% of those students who could not construct box plots passed.

(a) Construct a tree diagram of this scenario.

(b) Calculate the probability that a student is able to construct a box plot if it is known that he passed.

(a)

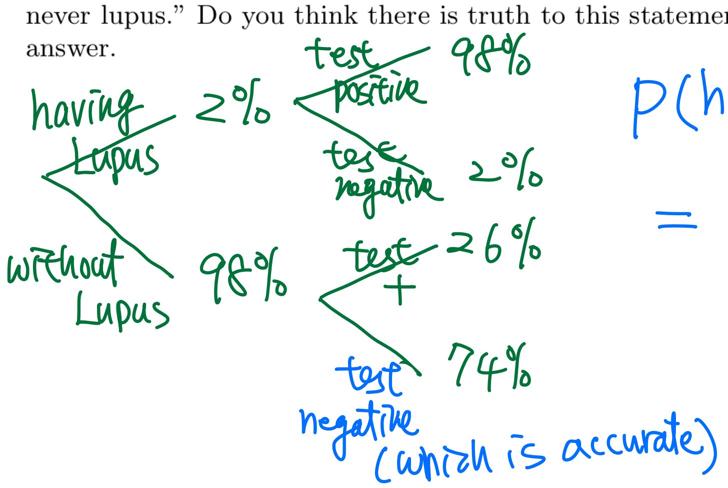



(b) P( construct a box plot | pass the class)

$$= \frac{P(\text{construt plot and pass})}{P(\text{contrut plot and pass}) + P(\text{not construt plot and pass})}$$

$$= \frac{80\% \cdot 86\%}{80\% \cdot 86\% + 20\% \cdot 65\%} = \frac{0.688}{0.818} \doteq 0.84$$

**3.21 It's never lupus.** Lupus is a medical phenomenon where antibodies that are supposed to attack foreign cells to prevent infections instead see plasma proteins as foreign bodies, leading to a high risk of blood clotting. It is believed that 2% of the population suffer from this disease. The test is 98% accurate if a person actually has the disease. The test is 74% accurate if a person does not have the disease. There is a line from the Fox television show *House* that is often used after a patient tests positive for lupus: "It's never lupus." Do you think there is truth to this statement? Use appropriate probabilities to support your answer.




P(having Lupus | test positive)

$$= \frac{P(\text{Lupus and Positive})}{P(\text{Lupus } \& \text{ p}) + P(\text{no Lupus } \& \text{ p})}$$

$$= \frac{0.98 \cdot 0.02}{0.98 \cdot 0.02 + 0.26 \cdot 0.98}$$

$$= \frac{0.02}{0.28} = 0.0714$$

7% is not much, so House might be right