Test2 Review, MAT1372

- Show all work and justify your answers and wishing you success.
- Useful formulas:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \qquad s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

$$\mu = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad s^2 = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n} \qquad s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$$

$$\mu = E(X) = \sum_{i=1}^{k} x_i P(X = x_i) \qquad \sigma^2 = \sum_{i=1}^{k}(x_i - \mu)^2 P(X = x_i) \qquad \sigma = \sqrt{\sum_{i=1}^{k}(x_i - \mu)^2 P(X = x_i)}$$

Uniform: $X \sim U(a,b)$; $\mu = \frac{a+b}{2}$; $\sigma = \sqrt{\frac{(b-a)^2}{12}}$

Bernoulli: $P(X = 1) = p, P(X = 0) = 1 - p$; $\mu = p$; $\sigma = \sqrt{p(1-p)}$

Binomial: $P(\text{exactly } k \text{ successes out of } n) = \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$; $\mu = np$; $\sigma = \sqrt{np(1-p)}$

Hypergeometric: $P(\text{pick } k \text{ from } r \mid \text{pick } n \text{ from } r + b) = P(X = k) = \frac{\binom{r}{k}\binom{b}{n-k}}{\binom{r+b}{n}}$; $\mu = \frac{nr}{r+b}$

Poisson: $P(\text{observe } k \text{ events}) = \frac{\lambda^k (2.718)^{-\lambda}}{k!}$; $e = 2.718$; $\mu = \lambda$; $\sigma = \sqrt{\lambda}$

1. A building contractor has sent in bids for three jobs. If the contractor obtains these jobs, they will yield respective profits of 20, 25, and 40 (in units of \$1000). On the other hand, for each job the contractor does not win, he will incur a loss (due to time and money already spent in making the bid) of 2 (in units of \$1000). If the probabilities that the contractor will get these jobs are, respectively, 0.3, 0.6, and 0.2, what is the expected total profit? (for example, let the first job be X_1, we have $P(X_1: win) = 0.3$ and $P(X_1: loss)$ =1-0.3= 0.7)

2. In the game of roulette, a wheel is spun and you place bets on where it will stop. One popular bet is that it will stop on a red slot; such a bet has an 18/38 chance of winning. If it stops on red, you double the money you bet. If not, you lose the money you bet. Suppose you play 3 times, each time with a \$1 bet. Let Y represent the total amount won or lost. Write a probability model for Y.

3. The distribution of passenger vehicle speeds traveling on the Interstate 5 Freeway (I-5) in California is nearly normal with a mean of 72.6 miles/hour and a standard deviation of 4.78 miles/hour.
(a) What percent of passenger vehicles travel slower than 80 miles/hour?
(b) What percent of passenger vehicles travel between 60 and 80 miles/hour?
(c) How fast do the fastest 5% of passenger vehicles travel?
(d) The speed limit on this stretch of the I-5 is 70 miles/hour. Approximate what percentage of the passenger

Please turn over and finish the rest of the question.

vehicles travel above the speed limit on this stretch of the I-5.

4. IQ examination scores for sixth-graders are normally distributed with mean value 100 and standard deviation 14.2. (a) What is the probability a randomly chosen sixth-grader has a score greater than 130?
   (b) What is the probability a randomly chosen sixth-grader has a score between 90 and 115?

5. Suppose that X follows $U(0, 2)$. Find (a) $P(X > \frac{1}{3})$, (b) $P(X \leq 1.4)$, (c) $P(0.6 < X \leq 1.8)$, (d) $P(X = 0.8)$, and (e) $P(2.2 < X < 2.4)$

6. Six **fair** coins are flipped. If the outcomes are independent, determine
   (a) the probability that there are a total of k heads, for k = 0, 1, 2, 3, 4, 5, 6. (Hint: Let X be a random variable which represents the number of heads when flipping 6 fair coins. Find $P(X = 0), P(X = 1), P(X = 2), P(X = 3), P(X = 4), P(X = 5)$, and $P(X = 6)$)
   (b) the expected value of the number of the heads.
   (c) the standard deviation of the number of the heads. (please keep the square root form as an answer)

7. Suppose a university announced that it admitted 2,500 students for the following year's freshman class. However, the university has dorm room spots for only 1,786 freshman students. If there is a 70% chance that an admitted student will decide to accept the offer and attend this university, what is the approximate probability that the university will not have enough dormitory room spots for the freshman class?

8. In the following problems, state whether the random variable X is binomial or hypergeometric. Also give its parameters (n and p if it is binomial or r, n and p if it is hypergeometric).

(1). A lot of 200 items contains 18 defectives. Let X denote the number of defectives in a sample of 20 items.

(2). A restaurant knows from past experience that 15 percent of all reservations do not show. Twenty reservations are expected tonight. Let X denote the number that show.

(3). In one version of the game of lotto each player selects six of the numbers from 1 to 54. The organizers also randomly select six of these numbers. These latter six are called the winning numbers. Let X denote how many of a given player's six selections are winning numbers.

(4). Each new fuse produced is independently defective with probability 0.05. Let X denote the number of defective fuses in the last 100 produced.

(5). Suppose that a collection of 100 fuses contains 5 that are defective. Let X denote the number of defectives discovered when 20 of them are randomly chosen and inspected.

9. A particular insurance company pays out an average of 4 major medical claims in a month.
   (a) Approximate the probability that it pays no major medical claims in the coming month?
   (b) Approximate the probability that it pays at most 2 major medical claims in the coming month?
   (c) Approximate the probability that it pays at least 4 major medical claims in the coming month?