

MAT1372, Quiz9, Fall2025

ID: _____

Name: _____

- This quiz consists of 1 question for a total of 10 points. You have 15 minutes to complete the quiz.
- Show all work and justify your answers.
- Wishing you success.
- Useful formulas:

Hypergeometric: $P(\text{pick } k \text{ from } r \mid \text{pick } n \text{ from } r + b) = P(X = k) = \frac{\binom{r}{k} \binom{b}{n-k}}{\binom{r+b}{n}}; \quad \mu = \frac{nr}{r+b}$

Poisson: $P(\text{observe } k \text{ events}) = \frac{\lambda^k e^{-\lambda}}{k!}; \quad e = 2.718; \quad \mu = \lambda; \quad \sigma = \sqrt{\lambda}$

1. A very skilled court stenographer makes one typographical error (typo) per hour on average.
 - (a) What probability distribution is most appropriate for calculating the probability of a given number of typos this stenographer makes in an hour?
 - (b) What are the mean and the standard deviation of the number of typos this stenographer makes?
 - (c) Would it be considered unusual if this stenographer made 4 typos in a given hour?
 - (d) Calculate the probability that this stenographer makes at most 2 typos in a given hour.

Sol (a) Poisson

(b) one typo per hour on average \Rightarrow mean $\lambda = 1$. standard deviation $\sigma = \sqrt{\lambda} = \sqrt{1} = 1$

(c) By checking how far away 4 typos are from the mean, we are able to know if it is unusual:

$$\frac{4-1}{1} = 3 \text{ which means 4 typos are } 3\sigma \text{ above the mean and it is unusual.}$$

(d) $P(\text{at most 2 typos})$

$$= P(\text{exactly 0 typos}) + P(\text{exactly 1 typo}) + P(\text{exactly 2 typos})$$

$$= \frac{1^0 \cdot e^{-1}}{0!} + \frac{1^1 \cdot e^{-1}}{1!} + \frac{1^2 \cdot e^{-1}}{2!} = e^{-1} \left(\frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} \right)$$

$$= \frac{1}{e} \left(1 + 1 + \frac{1}{2} \right) = \frac{1}{e} \cdot \frac{5}{2} = \frac{2.5}{e} = 0.919698 \dots$$