MAT1372, Quiz8, Fall2025

ID:_____ Name:_____

- This quiz consists of 1 question for a total of 10 points. You have 15 minutes to complete the quiz.
- Show all work and justify your answers.
- Wishing you success.
- Useful formulas:

Bernoulli: $P(X=1)=p, P(X=0)=1-p;$ $\mu=p;$ $\sigma=\sqrt{p(1-p)}$

Binomial: $P(\text{exactly } k \text{ successes out of } n)=\frac{n!}{k!(n-k)!}p^k(1-p)^{n-k};$ $\mu=np;$ $\sigma=\sqrt{np(1-p)}$

1. Six **fair** coins are flipped. If the outcomes are independent, determine
   (a) the probability that there are a total of k heads, for k = 0, 1, 2, 3, 4, 5, 6. (Hint: Let X be a random variable which represents the number of heads when flipping 6 fair coins. Find $P(X=0), P(X=1), P(X=2), P(X=3), P(X=4), P(X=5)$, and $P(X=6)$)
   (b) the expected value of the number of the heads.
   (c) the standard deviation of the number of the heads. (please keep the square root form as an answer)

(a) $P(X=0)=\frac{6!}{0!6!}\cdot\left(\frac{1}{2}\right)^0\left(1-\frac{1}{2}\right)^6=1\cdot 1\cdot\left(\frac{1}{2}\right)^6=\frac{1}{64}$

$n=6,\ p=\frac{1}{2}$ $k=0$ $0!=1$ $=6$

$k=1$ $P(X=1)=\frac{6!}{1!5!}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{6-1}=\frac{6}{1}\left(\frac{1}{2}\right)^6=\frac{6}{64}$

$k=2$ $P(X=2)=\frac{6!}{2!4!}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{6-2}=\frac{6\cdot 5}{2!}\left(\frac{1}{2}\right)^6=15\frac{1}{64}$

$k=3$ $P(X=3)=\frac{6!}{3!3!}\left(\frac{1}{2}\right)^3\left(1-\frac{1}{2}\right)^{6-3}=\frac{6\cdot 5\cdot 4}{6}\cdot\left(\frac{1}{2}\right)^3\cdot\left(\frac{1}{2}\right)^3=20\frac{1}{64}$

$k=4$ $P(X=4)=\frac{6!}{4!2!}\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^{6-4}=\frac{6\cdot 5}{2!}\left(\frac{1}{2}\right)^6=15\frac{1}{64}$

$k=5$ $P(X=5)=\frac{6!}{5!1!}\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{6-5}=\frac{6}{1!}\left(\frac{1}{2}\right)^6=6\frac{1}{64}$

$k=6$ $P(X=6)=\frac{6!}{6!0!}\left(\frac{1}{2}\right)^6\left(\frac{1}{2}\right)^{6-6}=1\cdot\left(\frac{1}{2}\right)^6=\frac{1}{64}$ $=1$

same

(b) $E(X)=0\cdot P(X=0)+1\cdot P(X=1)+2\cdot P(X=2)+3\cdot P(X=3)$

$+4+5+6\cdot P(X=6)=6\cdot\frac{1}{2}=3$

$n=6,\ p=\frac{1}{2}$

(c) $SD(X)=\sqrt{6\cdot\frac{1}{2}\cdot\frac{1}{2}}=\sqrt{\frac{3}{2}}$