MAT1372, Quiz2, Fall2025

ID:______________________________ Name: Sol.______________________________

- This quiz consists of 2 sets of questions for a total of 10 points.
- You have 15 minutes to complete the quiz.
- Show all work and justify your answers.
- Wishing you success.
- Useful formulas:

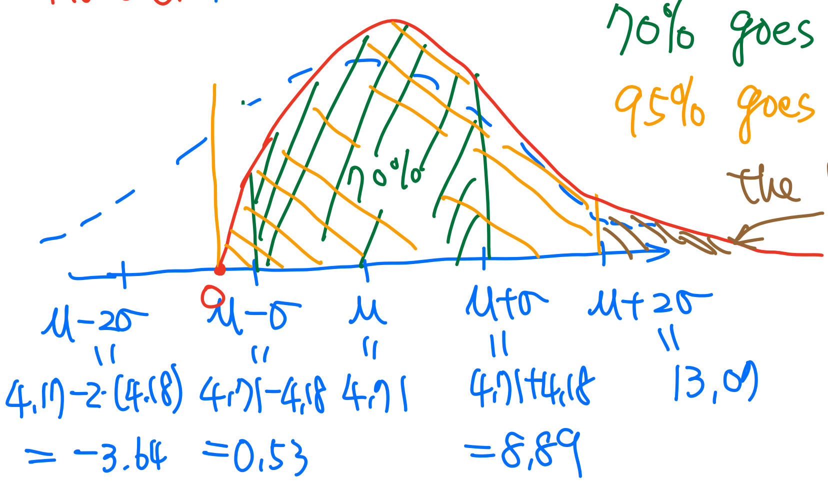
$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad s^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1} \qquad s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

$$\mu = \frac{x_1 + x_2 + \cdots + x_n}{n} \qquad s^2 = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n} \qquad s = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \mu)^2}{n}}$$

1. Students in an AP Statistics class were asked how many hours of television they watch per week (including online streaming). This sample yielded an average of 4.71 hours, with a standard deviation of 4.18 hours. Is the distribution of number of hours students watch television weekly symmetric? If not, what shape would you expect this distribution to have? Explain your reasoning.

Based on the concept of standard deviation σ, and mean μ.
70% of the data is between $\mu-\sigma$ and $\mu+\sigma$ and
95% of the data is between $\mu-2\sigma$ and $\mu+2\sigma$,

Now, let's make an observation: given $\mu=4.71$, $\sigma=4.18$,
then we were suppose to have the shape in blue.
However, since the minimum hour watching TV is 0, and then



70% goes from 0.53 to 8.89 and
95% goes from 0 to 13.07. and
the last 5% has to go to the right side
which makes it a right-skew distribution.

Please turn over and finish the rest of the question.

2. In a class of 25 students, 24 of them took an exam in class and 1 student took a make-up exam the following day. The professor graded the first batch of 24 exams and found an average score of 74 points with a standard deviation of 8.9 points. The student who took the make-up the following day scored 64 points on the exam.

(a) Without calculating the new mean, does the new student's score increase or decrease the average score? Why?

(b) What is the new average?

(c) Without calculating the new standard deviation, does the new student's score increase or decrease the standard deviation? Why?

(a) Since the old mean $\mu_{old} = 74$ and the make-up scored 64 is smaller than μ_{old}, so the new mean μ_{new} will be smaller.

(b) To find μ_{new}, we have $\mu_{old} = \frac{\text{sum of 24 scores}}{24}$ which implies the sum of 24 scores $= 24 \cdot \mu_{old} = 24 \cdot 74$

Then, we have $\mu_{new} = \frac{\text{sum of 24 scores} + 64 \text{(new score)}}{25}$

$$= \frac{24 \cdot 74 + 64}{25} = 73.6$$

(c) for standard deviation σ, there are 25 term of $(x_i - \mu_{new})^2$

① for the new score $(64 - 73.6)^2 = (9.6)^2 > (\sigma_{old})^2 = (8.9)^2$

② for the other 24 score

$$(x_i - 73.6)^2 - (x_i - 74)^2 = (2x_i - 147.6) \cdot (-73.6 + 74)$$

$$= (2x_i - 147.6) \cdot 0.4$$

sum of 24 score $= 24 \cdot \mu_{old}$

so $\sum_{i=1}^{24} \left((x_i - 73.6)^2 - (x_i - 74)^2 \right) = 0.4 \cdot \sum_{i=1}^{24} (2x_i - 147.6) = 0.4 \cdot \left[2 \boxed{\sum_{i=1}^{24} x_i} - 24 \cdot 147.6 \right]$

$$= 0.4 \cdot \left[2 \cdot 24 \cdot \mu_{old} - 24 \cdot 147.6 \right] = 0.4 \cdot 24 \left[2 \cdot \mu_{old} - 147.6 \right]$$

$$= 0.4 \cdot 24 \left[148 - 147.6 \right] > 0$$

End of this quiz.

Based on ①, ②, all 25 terms increase in σ_{new} so, $\sigma_{new} > \sigma_{old}$ (increase)