MAT1372, Classwork19, Fall2025

5.3 Hypothesis Testing of a Proportion (Conti.)

12. Significance Level in Hypothesis Testing. Significance level, denoted by α, is the probability of the study rejecting H_0, given H_0 is true. That is, if H_0 is true the significance level indicates how often the data lead us to incorrectly reject H_0

Example: If $\alpha = 0.05$, it means that for those cases where H_0 is true, we do not want to incorrectly reject H_0 more than 5% of the time

13. Confidence Interval versus Significance Level.

Using 95% confidence interval to evaluate a hypothesis is equivalent to significance level $\alpha = 0.05$

14. The p-value in Hypothesis Testing.

The p-value is probability of observing data at least as favorable to H_A as our current data set, if H_0 were true. We typically use a summary statistic of the data, in this section the sample proportion, to compute p-value

Example: In 11., the p-value is $p = 2 \times P(Z < -8.125) = 2 \cdot 0.0002 = 0.0004$

15. How should we evaluate the hypotheses in 11. using the p-value and standard significance level of $\alpha = 0.05$

If the null hypothesis were true, there's only an incredibly small chance of observing such an extreme deviation of $\hat{p}$ from 0.5. This means one of the following must be true:

(1) H_0 is true, and we just happened to observe something so extreme that it only happens about once in every 2500 times

(2) The H_A is true, which would be consistent with observing a sample proportion far away from 0.5

The first scenario is laughably improbable, while the second scenario seems much more plausible.

Formally, when we evaluate a hypothesis test, we compare p-value with α, which in this case $\alpha = 0.05$. Since "p-value $< \alpha$", we reject H_0

That is, the data provide **strong evidence against** H_0. The data indicate the direction of the difference: a majority of Americans do not support expanding the use of coal-powered energy.

16. Hypothesis Testing for a Single Proportion by Using Significance Level (α) and p-value

If a one-proportion hypothesis test is the correct procedure, there are four steps to completing the test:

Prepare. Identify the parameter of interest, list Hypothesis, identify α, and sample size n.

Check. get a $\hat{p}$ (from simple random sample) and calculat p-value. Verify the conditions to ensure p_0 (which is from H_0) is nearly Normal.

Calculate. If the condition holds, compute the SE, again using p_0, compute Z-score of $\hat{p}$, and identify p-value

Conclude. if $p < \alpha$: we reject H_0

if $p > \alpha$: we fail to reject H_0

17. Type 1 and Type 2 Error.

		Test conclusion	
		do not reject H_0	reject H_0 in favor of H_A
Truth	H_0 true	okay	**Type 1 Error**
	H_A true	**Type 2 Error**	okay

A Type 1 Error is rejecting Ho when Ho is true

A Type 2 Error is failing to reject Ho when Ho is false (H_A is true)

18. Choosing a Significance Level.

Traditional level is $\alpha = 0.05$. But We may select a level that is smaller or larger than 0.05 depending on consequences of any conclusions reached from the test

(1) If making a Type 1 Error is dangerous or especially costly, then choose a smaller α

But it increases the risk to make more Type 2 error.

(2) If a Type 2 Error is relatively more dangerous or much more costly than a Type 1 Error, then choose a larger α. But it increases the risk to make more Type I error

All in all, if we reduce how often we make one type of error, we generally make more of the other type.

6.1 Inference for a Single Proportion

1. Choosing a sample size n when estimating a proportion.

To set up a CI, we need a sample size, n, larger enough to make margin of error is sufficiently small that sample is useful. For example, find "n" so that the sample proportion is within ± 0.04 of the actual proportion in a 95% CI.

2. Example.

A university newspaper is conducting a survey to determine what fraction of students support a \$200 per year increase in fees to pay for a new football stadium. How big of a sample is required to ensure the margin of error is smaller than 0.04 using a 95% confidence level?

The margin of error for a sample proportion $z^* \times SE = z^* \cdot \sqrt{\frac{p(1-p)}{n}}$

For a 95% CI, we have $z^* = 1.96$: $1.96 \times \sqrt{\frac{p(1-p)}{n}} < 0.04$

Two unknown: p or n:

For p, we can either use the p from a prior survey or use the worst case value which is $p = 0.5$

Then we have $1.96 \cdot \sqrt{\frac{0.5(1-0.5)}{n}} < 0.04$

$\Rightarrow \left(\sqrt{\frac{0.5(1-0.5)}{n}}\right)^2 < \left(\frac{0.04}{1.96}\right)^2 \Rightarrow \frac{0.25}{n} < \left(\frac{0.04}{1.96}\right)^2 \Rightarrow n > 0.25 \cdot \left(\frac{1.96}{0.04}\right)^2$

$= 600.25$

We need over 600.25 participants (or 601 people) to build a 95% CI with ± 0.04 from sample proportion