MAT1372, Classwork14, Fall2025

$\binom{5}{2} = \frac{5!}{2!\,3!}$

${}_5C_2$

Hypergeometric Distribution(QR Code5)

1. A committee of 4 people is to be selected from a group of 5 men and 7 women. If the selection is made randomly, what is the probability the committee will consist of 2 men and 2 women?

$P(2m, 2W \mid 4 \text{ ppl out of } 12) = \frac{\binom{5}{2}\binom{7}{2}}{\binom{12}{4}} = 0.3939$

2. Hypergeometric Random Variable.

How to describe a hypergeometric random variable in word?

It describes the number of successes in a fixed number of trails without replacement.

Example: The classical application of the hypergeometric distribution is sampling w/o replacement.

How to describe a hypergeometric random variable in math?

It describes the probability of k successes in n draws w/o replacement

Example: Find the probability of the outcome of drawing k green marbles out of r total green marbles, and draw n−k red marbles out of b red marbles, in n rounds

3. Definition of the Hypergeometric Distribution.

Suppose X is a R.V. with a Hypergeometric distri, denoted by

$X \sim H(r, b, n)$

The probability of exactly k observation be selected from r observation (the size of 1st group) given n (the size of the chosen sample) be picked from r+b

$P(\text{exactly } k \text{ from } r \mid \text{picked } n \text{ from } r+b) = P(X = k) = \frac{\binom{r}{k}\binom{b}{n-k}}{\binom{r+b}{n}}$

with its mean, variance, and standard deviation of the number of observed successes

$$\mu = \frac{nr}{r+b}, \qquad \sigma^2 = n\frac{r}{r+b}\left(1-\frac{r}{r+b}\right)\frac{r+b-n}{r+b-1} \qquad \sigma = \sqrt{n\frac{r}{r+b}\left(1-\frac{r}{r+b}\right)\frac{r+b-n}{r+b-1}}$$

(If $N = r+b$ and $p = \frac{r}{r+b} = \frac{r}{N}$, then $X \sim H(N, r, n)$, $\mu = np = n\frac{r}{r+b}$

$\sigma^2 = n \cdot p(1-p) \cdot \frac{N-n}{N-1}$

4. A school site committee is to be chosen randomly from 6 men and 5 women. If the committee consists of 4 members chosen randomly, what is the probability that 2 of them are women? How many women do you expect to be on the committee?

$P(\text{exactly 2 women}) = \frac{\binom{5}{2}\binom{6}{2}}{\binom{11}{4}} = 0.4545.$ $\mu = 4 \cdot \frac{5}{5+6} = \frac{20}{11} = 1.81$

$n=4, r=5, b=6$

$H(r, b, n)$

5. Let $X \sim H(r, b, 1)$. (a) Find $P(X = 0)$ and $P(X = 1)$. (b) Do we have $P(X = 2)$? Or $P(X = k)$ for $k > 2$?

$n=1$

(a) $P(X=0) = \frac{\binom{r}{0}\binom{b}{1}}{\binom{r+b}{1}} = \frac{1 \cdot b}{r+b} = 1-p$, $P(X=1) = \frac{\binom{r}{1}\binom{b}{0}}{\binom{r+b}{1}} = \frac{r \cdot 1}{r+b} = p$

$k=0$ $\quad k=1$

$\Rightarrow X$ follows Bernoulli distribution

$P(X=2)$

$k=2>n$

4.5 Poisson Distribution

1. Poisson Distribution.

How to describe a Poisson distribution in word?

The Poisson distribution helps us describe the number of such events that will occur in a day for a fixed population if the individual within the population are independent. The Poisson distribution could also be used over another unit of time, such as an hour or a week.

Example: The people have heart attacks per day, the people get married per month.

How to describe a Poisson distribution in math?

The Poisson distribution is a discrete distribution that expresses the probability of a given number of events occuring in a fixed interval of time if these events occur with a known constant mean rate and independent of the time since the last event.

Example: Consider a call center which receives an average 3 calls per mins

2. If a call center receives 20 calls between 8 a.m. and 12 p.m. What is the average calling this center gets in 15 minutes? 4 hours = 4·4 15min = 16 (15mins) (8~12pm) $\text{average} = \frac{20}{16} = 1.25$ call

3. Definition of the Poisson Distribution.

Suppose we are watching for events and the number of observed events follows a Poisson distribution with rate λ (lambda). Then

$$P(\text{observe } k \text{ events}) = \frac{\lambda^k \cdot e^{-\lambda}}{k!}$$

where k may take a value 0, 1, 2, and so on until "∞". $e \approx 2.718$

The mean and standard deviation of this distribution are

$$\mu = \underline{\lambda}, \qquad \sigma = \underline{\sqrt{\lambda}}.$$

4. Tom receives about 6 telephone calls between 8 a.m. and 10 a.m. What is the probability that Tom receives more than one call in the next 15 minutes?

$\lambda = \text{average} = \frac{6}{8} = 0.75$, X = the number of calls Tom got per 15 mins

got 2 calls got 3 calls

$$P(X > 1) = P(X=2) + P(X=3) + P(X=4) + \cdots + P(X = \infty)$$

$$= 1 - P(X=0) - P(X=1)$$

$$= 1 - \frac{(0.75)^0 \cdot e^{-0.75}}{0!} - \frac{(0.75)^1 \cdot e^{-0.75}}{1!}$$

$$= 1 - \frac{1 \cdot e^{-0.75}}{1} - \frac{0.75 \cdot e^{-0.75}}{1}$$

$$= 1 - 0.4724 - 0.3543 = 0.1733$$