MAT1372, Classwork12, Fall2025

4.1 Normal Distribution

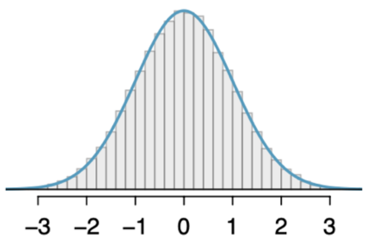
1. Normal Distribution Model.

How to describe a normal distribution in word? A symmetric, unimodal, bell-shape curve.

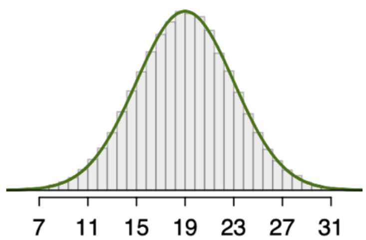
How to describe a normal distribution in math? The distribution's parameters: mean and standard deviation

The notation of a normal distribution with mean μ and standard deviation σ: $N(\mu, \sigma)$.

2. Write down the shorthand for the normal distributions with the given graphs:



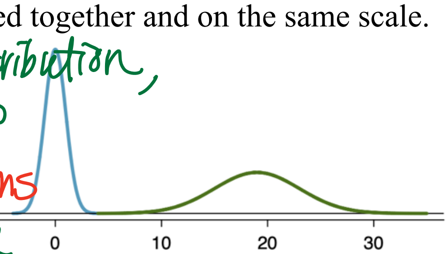
(a) The mean for this distribution is $\mu=0$, the standard deivation is $\sigma=1$. This is called standard Normal distribution and it is written as $N(0,1)$



(b) $\mu=19$, $\sigma=19-15=4$. This is called a Normal distribation and it is writen as $N(19,4)$

3. The given figure is the normal distributions shown in 2.(a) and (b) but plotted together and on the same scale.

What do you observe? Even though they are both normal disstribution, it looks very different. We need to put data onto a standardlized scale, which can make comparisons more reasonable



4. Standardizing with Z-scores.

Description: Z-score is a standardization technique most commonly employed for nearly normal observation, but that may be used with any distribution. The Z-score of an observation is defined as the number of σ it falls above or below the mean

Definition: If x is an observation form a distribution $N(\mu,\sigma)$, we define Z-score as

$$Z = \frac{x-\mu}{\sigma}$$

Example. $Z = 1$: the observation x is positive one standard deviation above from the mean.

$Z = -1.5$: the observation is 1.5 standard deviation below from mean.

5. Given table shows the mean and standard deviation for total scores on the SAT and ACT. The distribution of SAT and ACT scores are both nearly normal. Suppose Ann scored 1300 on her SAT and Tom scored 24 on his ACT.

	SAT	ACT
μ: Mean	1100	21
σ: SD	200	6

(a) Use Tom's ACT score along with the ACT mean and standard deviation to find his Z-score.

$\mu=21$, $\sigma=6$.

$$Z_{Tom} = \frac{\text{Tom's score} - \mu}{\sigma} = \frac{24-21}{6} = \frac{3}{6} = 0.5$$

Tom's score is $\frac{1}{2}$ standard deviation above the mean

(b) Use Ann's SAT score along with the SAT mean and standard deviation to find her Z-score.

$\mu = 1100, \ \sigma = 200$

$Z_{Ann} = \frac{\text{Ann's Score} - \mu}{\sigma} = \frac{1300 - 1100}{200} = \frac{200}{200} = 1$

(c) Who performed better?

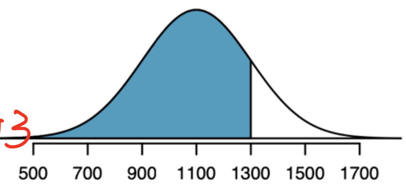
Ann has a better performence since $Z_{Ann} > Z_{Tom}$

6. The Application of Z-score.

We can use Z-score to roughly identify which observation are more unusual than others.

Assume x_1 has Z-score z_1, x_2 has Z-score z_2.
If $|z_1| > |z_2|$, then x_1 is more unusual than x_2

7. Finding tail areas by the probability table.

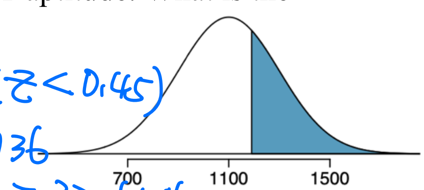


(a) What fraction of people have an SAT score below Ann's score of 1300?

$Z_{Ann} = \frac{1300 - 1100}{200} = 1$, $P(Z < Z_{Ann}) = P(Z < 1) = 0.8413$

(b) Shannon is a randomly selected SAT taker, and nothing is known about her SAT aptitude. What is the probability her scores at least 1190 on her SATs? $Z_S = \frac{1190 - 1100}{200} = 0.45$

$P(\text{at least } 1190) = P(\text{Score} \geq 1190) = P(Z \geq 0.45) = 1 - P(Z < 0.45)$
$= 1 - 0.6736$
$= 32.64\%$



8. Finding Areas To the Right:

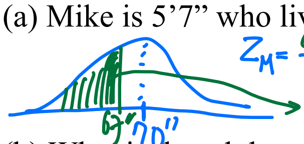
If you would like the area to the right, first find the area to left and then subtract this number from 1

9. ALWAYS DRAW A PICTURE FIRST, AND FIND THE Z-SCORE SECOND.

For any normal probability situation, always draw and label the normal curve and shade the area of interest first. After that, identify the Z-score for the value of interest.

10. Given the heights of male adults in the US is nearly normal which follows $N(\underset{\mu}{70.0"}, \underset{\sigma}{3.3"})$.

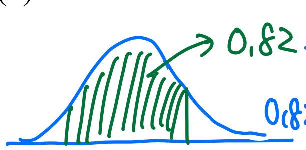
(a) Mike is 5'7" who lives in the US. What is Mike's height percentile? $5'7" = 5 \cdot 12" + 7" = 67"$



$Z_M = \frac{67 - 70}{3.3} = -0.91$

$P(Z > Z_M) = P(Z > -0.91) = 0.1814, \Rightarrow 18.14\%$

(b) What is the adult male height at the 82nd percentile? Let the heigh be "h" (inches)



$Z_m = \frac{h - 70}{3.3}$

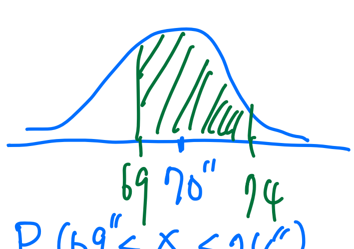
$0.82 = P(Z > Z_m) = P(Z > \frac{h-70}{3.3})$

$\frac{h - 70}{3.3} = 0.92$

$h = 0.92 \cdot 3.3 + 70 = 73.04" = 6'1"$

(c) What is the probability that a random adult male is between 5'9" and 6'2"?

69" 74"



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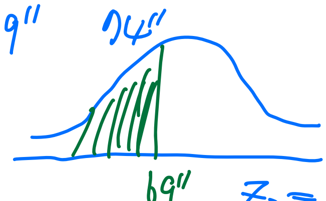

$P(69" < X < 74")$

$Z_1 = \frac{74 - 70}{3.3} = 1.21$

$Z_2 = \frac{69 - 70}{3.3} = -0.3$

$= P(-0.3 < Z < 1.21) = P(Z < 1.21) - P(Z < -0.3) = 0.8869 - 0.3821 = 0.5048$