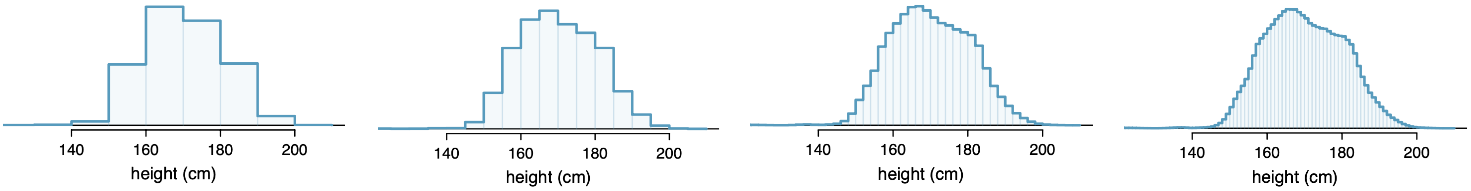
MAT1372, Classwork11, Fall2025

3.5 Continuous Distributions

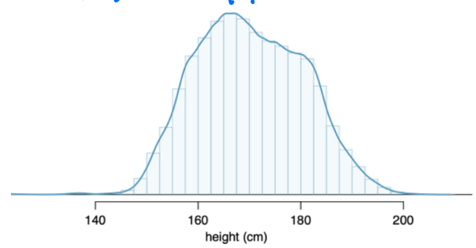
1. From histograms (discrete distributions) to continuous distributions.



Given figure shows a few different hollow histograms for the heights of US adults.

(a) How does changing the number of bins allow you to make different interpretations of the data?

Adding more bins provide greater detail. This sample is extremely large, which is why much smaller bins still work well.



(b) When the number of bins gets more and more, the width of each bin

gets smaller and we will get a Smooth curve eventually

which represents a continuous probability distribution

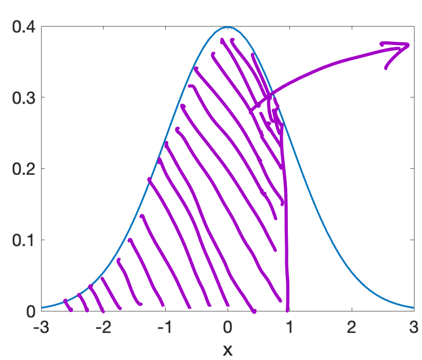
2. The Continuous Probability Distribution and Probability Density Function (pdf).

The graph of a continuous probability distribution is a curve. The curve is called Probability Density Function (pdf) or density (function). The property of density function: the total area under the curve is 1

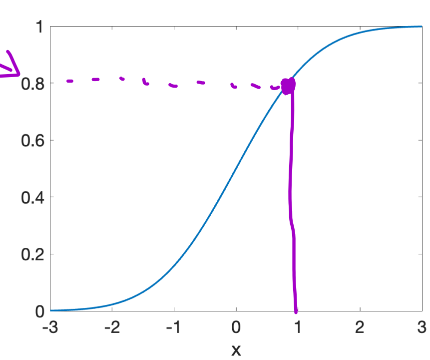
3. The Cumulative Distribution Function (cdf)

Area under the curve is given a different function called Cumulative distribution function (cdf). Cdf is used to evaluate probability as area.

4. Example of a Probability Density Function and its Cumulative Distribution Function.



area =



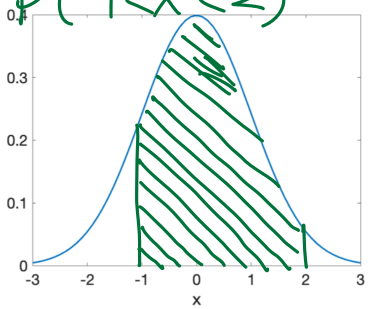
The pdf of the normal distribution with $\mu = 0, \sigma = 1$

The cdf of the given pdf of the normal distribution

The probability $P(x<1)$ in pdf is the value in cdf when $x = 1$

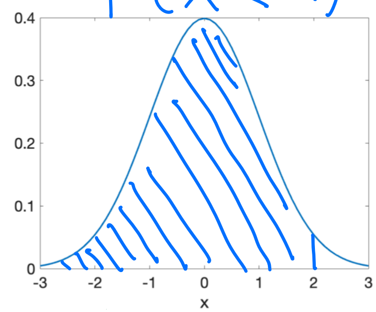
The probaility $P(-1<X<2)$ in pdf is ....

$P(-1<X<2)$



$=$

$P(X<2)$



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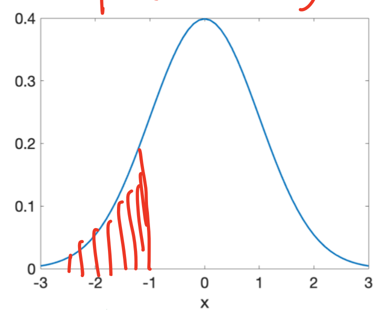
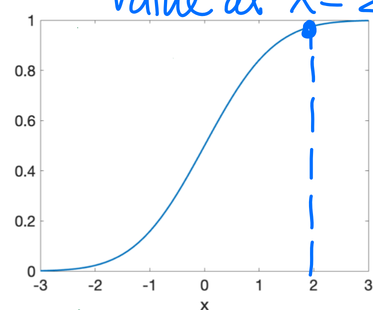
$P(X<-1)$



$=$

Value at $x=2$



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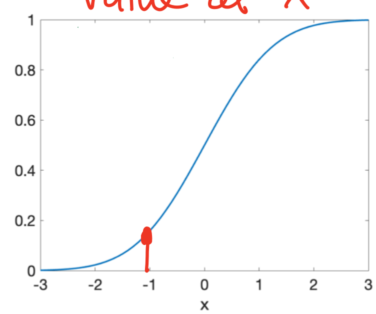
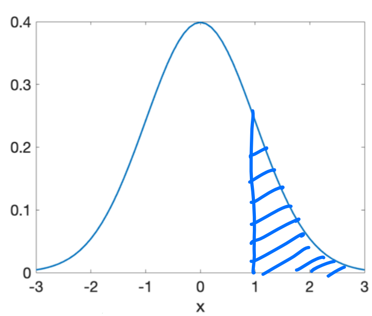
value at $x=-1$



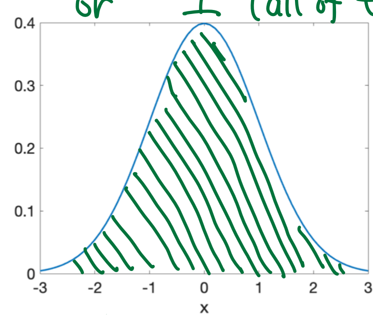
... is the difference of the values of the cdf at $x=2$ and $x=-1$

$P(X>1)$ in pdf is ....



$=$

$P(-\infty<x<\infty)$ or 1 (all of them)



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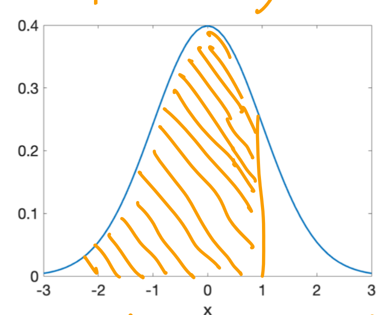
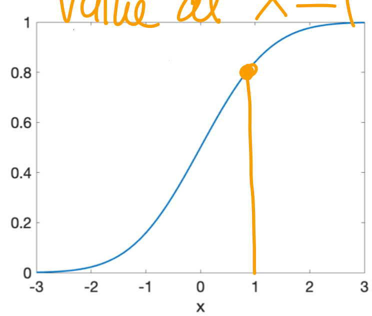
$P(X<1)$



$= \quad 1 \quad -$

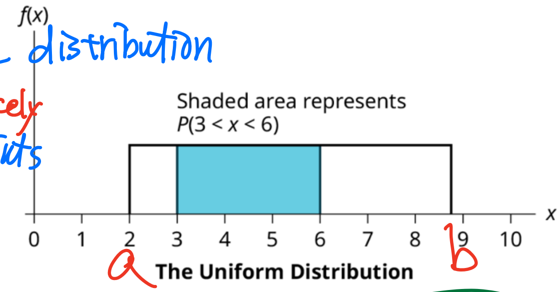
value at $X=1$



.... 1 minus the cdf value at $x=1$.

5. The Uniform Distribution.

The uniform distribution is a continuous probability distribution and is concerned with events that are equally likely to occur. Be Careful to notice if the endpoints are exclusive or inclusive in the data.



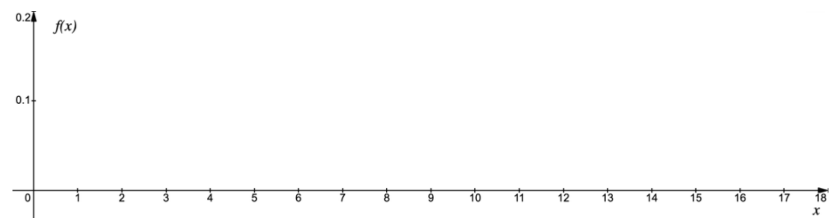
$X \sim U(a, b)$: the random Variable X follows a Uniform distribution from a to b, $\mu = \dfrac{b+a}{2}$, $\sigma = \sqrt{\dfrac{(b-a)^2}{12}}$

two endpoints

6. Suppose the time it takes a student to finish a quiz is uniformly distributed between 6 and 15 minutes, inclusive. Let X = the time, in minutes, it takes a student to finish a quiz. Then $X \sim U(a, b)$.

(a) Find a and b and describe what they represent.

(b) Write the distribution and graph it.



(c) Find the probability that a randomly selected student needs at least 8 minutes to complete the quiz.

(d) Find the probability that a student needs at least 8 minutes to finish the quiz given that this individual has already taken more than 7 minutes.

(e) Find the 30th percentile of quiz finishing minutes.

(f) Find the mean μ and standard deviation σ.

7. Sum up the properties of Probability Density Function in general.

(1) The entire area under ____________________ and above ____________________ is equal to _______

(2) Probability is found for ____________________ of x values *rather than* for individual x values.

- $P(c < x < d)$ is the probability that ____.
- $P(c < x < d)$ is the area ____.
- $P(c < x < d)$ is ______________ as $P(c \leq x \leq d)$ because ____________________________________.
- $P(x = c) =$ ________: The probability that x takes on ________________________________ is _____.

(why?)