MAT1372, Classwork10, Fall2025

3.4 Random variables

1. Two books are assigned for a statistics class: a textbook and its corresponding study guide. The university bookstore determined 20% of enrolled students do not buy either book, 55% buy the textbook only, and 25% buy both books, and these percentages are relatively constant from one term to another.

(a) If there are 100 students enrolled, how many books should the bookstore expect to sell to this class?

$100 \times 20\% = 20$ buy no books — 0
$100 \times 55\% = 55$ buy 1 book — 55×1
$100 \times 25\% = 25$ buy 2 books — 25×2
$\Rightarrow 0 + 55 + 50 = 105$ books

(b) The textbook costs \$137 and the study guide \$33. How much revenue should the bookstore expect from this class of 100 students?

$20 \cdot \$0 + 55 \cdot \$137 + 25 \cdot \$\underbrace{170}_{(137+33)} = \$11785$

(c) What is the average revenue per student for this course?

The average is $\frac{\$11785}{100} = \$117.85$

2. Random Variable.

We call a variable with a numerical outcome a random variable. We represent this random variable with capital letters, X, Y, Z

3. In 1., we assume that the amount of money a single student will spend on the statistics books is a random variable, and we represent it by X.

i	x_i	$P(X = x_i)$	$x_i \times P(X = x_i)$
1	0	0.2	0
2	137	0.55	75.35
3	170	0.25	42.5
T		1.0	

(a) What are the possible outcomes of X?

$X = 0$, $X = 137$, $X = 170$

(b) What are the probability for each outcome of X?

$P(X=0) = 0.2$, $P(X=137) = 0.55$, $P(X=170) = 0.25$

4. Expected Value of a Discrete Random Variable.

If X takes outcomes $x_1, x_2, \ldots, x_k$ with probabilities $P(X=x_1), P(X=x_2), \ldots, P(X=x_k)$, the expected value of X is

$$E(X) = x_1 \cdot P(X=x_1) + x_2 \cdot P(X=x_2) + \cdots + x_k \cdot P(X=x_k)$$
$$= \sum_{i=1}^{k} x_i \cdot P(X=x_i), \quad \mu = E(X)$$

5. What is the expected value of the case in 3.?

$$E(X) = 0 \cdot P(X=0) + 137 \cdot P(X=137) + 170 \cdot P(X=170)$$
$$0 + 75.35 + 42.5 = 117.85$$

6. Why is the result in 5. (the expected value) the same as the result in 1.(c) (the mean average revenue)?

Based on the definition of expected value of a random variable, it is a weighted mean and the expected value represents the average outcome of this random variable.

7. General Variance Formula

If X takes outcomes $x_1, \cdots, x_k$ with probabilities $P(X = x_1), \cdots, P(X = x_k)$, and the expected value $\mu = E(X)$,

Then the variance of X, denoted by $Var(X)$ or the symbol σ^2, is

$$Var(X) = (x_1-\mu)^2 \cdot P(X=x_1) + (x_2-\mu)^2 \cdot P(X=x_2) + \cdots + (x_k-\mu)^2 \cdot P(X=x_k)$$
$$= \sum_{i=1}^{k} (x_i-\mu)^2 \cdot P(X=x_i)$$

The Standard deviation of X, labeled σ, is the square root of the variance.

8. Find the variance of X and the standard deviation of X in 3.?

i	x_i	$P(X = x_i)$	$x_i \times P(X = x_i)$	$x_i - \mu$	$(x_i - \mu)^2$	$(x_i - \mu)^2 \times P(X = x_i)$
1	\$0	0.2	0	−117.85	13888.62	2777.7
2	\$137	0.55	75.35	19.15	366.72	201.7
3	\$170	0.25	42.50	52.15	2719.62	+ 679.9
Total		1.0	$\mu =$ 117.85 $E(X)$			3659.3

$Var(X) =$ 3659.3 and $\sigma = \sqrt{Var(X)} = \sqrt{3659.3} = 60.49$

9. Linear Combinations of Random Variables, the Average Result, and the Variability

Let X and Y be random variables. Then, for two fixed numbers a, and b, we have

(1) A linear combinations of X and Y: $aX + bY$ is also a random variable

(2) The Expected Value of $aX + bY$: $E(aX+bY) = a \cdot E(X) + b \cdot E(Y)$

(3) The Variance for $aX + bY$: $Var(aX+bY) = a^2 Var(X) + b^2 Var(Y)$

This equation is valid as long as the random variables X and Y are independent of each other.

10. Leonard has invested \$6000 in Caterpillar Inc (stock ticker: CAT) and \$2000 in Exxon Mobil Corp (XOM). If X represents the change in Caterpillar's stock next month and Y represents the change in Exxon Mobil's stock next month, then

(1) write an equation that describes how much money will be made or lost in Leonard's stocks for the month.

$6000 \cdot X + 2000 \cdot Y$

(2) Given the mean, standard deviation, and variance of the CAT and XOM stocks in the table. Find the expected change, the variance, and the standard deviation in Leonard's stock portfolio for next month.

	Mean ($\bar{x}$)	Standard deviation (s)	Variance (s^2)
$E(X)$ CAT	0.0204	0.0757	0.0057
$E(Y)$ XOM	0.0025	0.0455	0.0021

$E(6000 \cdot X + 2000 \cdot Y) =$

$6000 \cdot E(X) + 2000 \cdot E(Y) = 6000 \cdot 0.0204 + 2000 \cdot 0.0025 = \$127.4$

$Var(6000 \cdot X + 2000 \cdot Y) = 6000^2 \, Var(X) + 2000^2 \cdot Var(Y) = 213600$

$\sigma = \sqrt{Var} = \sqrt{213600} = 462.168$

(3) You should have found that Leonard expects a positive gain in (2). However, would you be surprised if he actually had a loss this month?

NO. While stocks tend to rise over time, they are often volatile in the short term