MAT1372, Classwork9, Fall2025

3.3 Sampling from a small population & Counting Techniques(QR Code3)

1. Sampling with/without replacement

Sampling with replacement: selected subjects are put back into the population before another subject are sampled.

Sampling without replacement: Selected subjects will not be in the "pool" for selection

2. Professors sometimes select a student at random to answer a question and each student has an equal chance of being selected and there are 10 people in your class. If the professor asks 3 questions and assume that one **will not** be picked twice in a given lecture, then

(a) what is the probability that you will not be selected?

$P(Q_1 = \text{Not picked}, Q_2 = \text{not picked}, Q_3 = \text{not picked}) = \frac{9}{10}\cdot\frac{8}{9}\cdot\frac{7}{8} = \frac{7}{10}$

$= P(Q_1 = \text{not picked}) \cdot P(Q_2 = \text{not picked given } Q_1 = \text{not picked}) \cdot P(Q_3 = \text{not picked} \mid Q_1, Q_2 = \text{not picked})$

(b) what is the probability that you will be selected for the first question? $P(Q_1 = \text{picked}) = \frac{1}{10}$

(c) what is the probability that you will be selected for the second question?

$P(Q_1 = \text{not picked and } Q_2 = \text{picked}) = \frac{9}{10}\cdot\frac{1}{9} = \frac{1}{10}$

not independent

fair chance to get picked

(d) what is the probability that you will be selected for the third question?

$P(Q_1 = \text{not picked}, Q_2 = \text{not picked}, Q_3 = \text{picked}) = \frac{9}{10}\cdot\frac{8}{9}\cdot\frac{1}{8} = \frac{1}{10}$

(e) what is the probability that you will be selected for one question?

(b)+(c)+(d) $= \frac{3}{10}$ or $1-(a) = 1-\frac{7}{10} = \frac{3}{10} = 30\%$

3. In 2., If the professor asks 3 questions and assume that one **could be** picked more than twice in a given lecture, then

(a) what is the probability that you will not be selected?

$P(Q_1 = \text{not picked}, Q_2 = \text{not picked}, Q_3 = \text{not picked}) = \frac{9}{10}\cdot\frac{9}{10}\cdot\frac{9}{10} = \frac{729}{1000}$

(b) what is the probability that you will be selected for the first question? $P(Q_1 = \text{picked}) = \frac{1}{10}$.

(c) what is the probability that you will be selected for the second question?

$P(Q_1 = \text{not picked}, Q_2 = \text{picked}) = \frac{9}{10}\cdot\frac{1}{10} = \frac{9}{100}$

(d) what is the probability that you will be selected for the third question?

$P(Q_1 = \text{not picked}, Q_2 = \text{not picked}, Q_3 = \text{picked}) = \frac{9}{10}\cdot\frac{9}{10}\cdot\frac{1}{10} = \frac{81}{1000}$

(e) what is the probability that you will be selected for at least one question?

(b)+(c)+(d) or $1-(a) = \frac{271}{1000} = 27.1\%$

4. In 2., assume there are **100 students** and the professor asks 3 questions.

(a) If one **will not** be picked twice then what is the probability that you will be selected for one question?

$1-P(Q_1=\text{not picked}, Q_2=\text{not picked}, Q_3=\text{not picked})=1-\frac{99}{100}\cdot\frac{98}{99}\cdot\frac{97}{98}=1-\frac{97}{100}=\frac{3}{100}=3\%$

(b) If one **could** be picked twice then what is the probability that you will be selected for at least one question?

$1-P(Q_1=\text{not picked}, Q_2=\text{not picked}, Q_3=\text{not picked})=1-\frac{99}{100}\cdot\frac{99}{100}\cdot\frac{99}{100}=\frac{2.9701}{100}=2.97\%$

5. What can you observe the difference of the results once the sample space getting larger?

In 2.3 30% (without replacement) 3%

27.1% (with replacement) 2.97%

10 → 100 students = sample size

6. How many three letter "words" can be made from the letters a, b, and c with **no letters repeating**? A "word" is just an ordered group of letters. It doesn't have to be a real word in a dictionary.

abc bac cab
acb bca cba

6 combination

$3 \times 2 \times 1 = 6$ combination

(3 options, 2 options, 1 option)

7. Factorial $n!$: $n! = n\cdot(n-1)\cdot(n-2)\cdot(n-3)\cdot\cdots\cdot 2\cdot 1$

8. How many three letter "words" can be made from the letters a, b,…, z with **no letters repeating**?

$26 \times 25 \times 24 = {}_{26}P_3 = \frac{26!}{(26-3)!} = \frac{26!}{23!}$

$= \frac{26\times25\times24\times23\times\cdots\times2\times1}{23\times22\times21\times\cdots\times2\times1} = 26\times25\times24$

(26 options, 25 options, 24 options)

9. Permutation Formula ${}_nP_r$: ${}_nP_r = \frac{n!}{(n-r)!}$ (picked r objects from n objects when order matters)

10. How many ways you choose three letters from the letters a, b,…, z with **no letters repeating**?

$\frac{26\cdot25\cdot24}{6} = {}_{26}C_3 = \frac{26!}{3!\,(26-3)!}$

(26 options, 25 options, 24 options)

11. Combination Formula ${}_nC_r$: ${}_nC_r = \frac{n!}{r!\,(n-r)!}$ (picked r objects from n objects when order doesn't matter)

12. How many three letter "words" can be made from the letters a, b, and b?

a b b, b a b, b b a
a b b, b a b, b b a

$\frac{3\cdot2\cdot1}{2\cdot1} = \frac{3!}{2!}$ ← How many times b repeat?

13. How many six letter "words" can be made from the letters a, b, b, c, c, and c?

$\frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{2!\,3!} = 60$

"c" repeats 3 times

"b" repeats 2 times