MAT1372, Classwork8, Fall2025

3.2 Conditional Probability (Conti.)

6. (a) Write out the following statement in conditional probability notation: "The probability that the ML prediction was correct, if the photo was about fashion" and find the probability.

$P(\text{ML pre\_fashion given truth is fashion}) = \frac{197}{309}$

(b) Determine the probability that the algorithm is incorrect if it is known the photo is about fashion.

$P(\text{ML pre\_not give truth is fashion}) = \frac{112}{309}$

(c) Compute $P(\text{mach\_learn is pred\_fashion} \mid \text{truth is fashion}) + P(\text{mach\_learn is pred\_not} \mid \text{truth is fashion})$.

$\frac{197}{309} + \frac{112}{309} = \frac{309}{309} = 1$

7. SUM OF CONDITIONAL PROBABILITIES:

Let $A_1, A_2, \dots, A_K$ represent all the disjoint outcomes for a variable, Then if B is an event, possibly for another variable, we have

$$P(A_1 \mid B) + P(A_2 \mid B) + \dots + P(A_K \mid B) = 1$$

The rule for complements holds $P(A \mid B) = 1 - P(A^c \mid B)$

8. **Independence** considerations in conditional probability

Let X and Y represent the outcomes of rolling two dice.

(a) What is the probability that the first die, X, is 1? $P(X=1) = \frac{1}{6}$

(b) What is the probability that both X and Y are 1? $P(X=1 \text{ and } Y=1) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$ $P(X=1) \cdot P(Y=1)$

(c) Use the formula for conditional probability to compute P (Y = 1 | X = 1).

(d) What is P (Y = 1)? Is this different from the answer from part (c)? Explain.

(c) $P(Y=1 \mid X=1) = \frac{P(Y=1 \text{ and } X=1)}{P(X=1)} = \frac{P(Y=1) \cdot P(X=1)}{P(X=1)} = P(Y=1)$

(d) $P(Y=1) = \frac{1}{6}$. Since event "$X=1$" and "$Y=1$" are independent.

9. GENERAL MULTIPLICATION RULE:

If A and B are two outcomes, then

$$P(A \text{ and } B) = P(A \mid B) \times P(B)$$

10. The smallpox data set provides a sample of 6,224 individuals and each case represents one person with two variables: inoculated (yes or no) and result (lived or died).

Suppose we are given only two pieces of information: 96.08% of residents were not inoculated, and 85.88% of the residents who were not inoculated ended up surviving. How could we compute the probability that a resident was not inoculated and lived?

① ②

$P(\text{inoculated} = \text{NO and lived}) = P(\text{inoculated} = \text{NO}) \times P(\text{lived given inoculated} = \text{NO})$

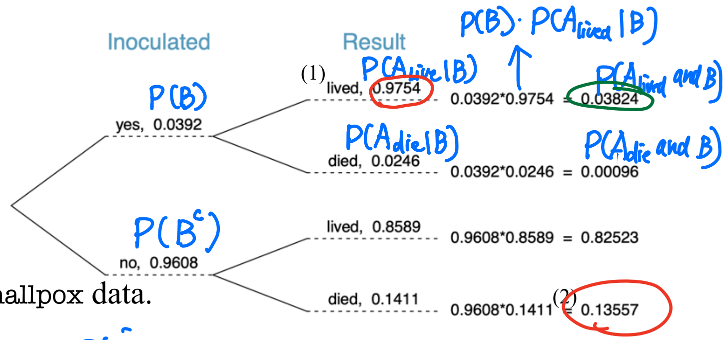
$= 96.08\% \cdot 85.88\%$

$= 82.51\%$

11. Tree Diagram

		inoculated		
		yes	no	Total
result	lived	0.0382	0.8252	0.8634
	died	0.0010	0.1356	0.1366
	Total	0.0392	0.9608	1.0000

Figure 3.16: Table proportions for the `smallpox` data, computed by dividing each count by the table total, 6224.



Given the contingency table and the tree diagram about the smallpox data.

In the tree diagram, the first branch for inoculation is said to be the primary branch while the other branches are secondary.

(a) Describe the number 0.9754 in (1) from the tree diagram in conditional probability.

$P(\text{result}=\text{lived} \mid \text{inoculated}=\text{Yes})$

(b) Describe the number 0.13557 in (2) from the tree diagram in conditional probability.

$P(\text{result}=\text{died} \text{ and } \text{inoculated}=\text{NO})$

(c) From the tree diagram, how to find the probability about the people who lived?

$P(\text{lived}) = P(\text{result}=\text{lived} \text{ and } \text{inoculated}=\text{Yes}) + P(\text{result}=\text{lived} \text{ and } \text{inoculated}=\text{NO})$

(d) Can we find the probability that people got inoculated if they lived? $= 0.03824 + 0.8252 = 0.8634$

$$P(\text{inoculated}=\text{Yes} \mid \text{lived}=\text{Yes}) = \frac{P(\text{inoculated}=\text{Yes} \text{ and } \text{lived}=\text{Yes})}{P(\text{lived})} = \frac{0.0382}{0.8634}$$

12. BAYES' THEOREM: INVERTING PROBABILITIES:

Consider the following conditional probability for variable 1 and variable 2:

P (outcome A_1 of variable 1 | outcome B of variable 2)

Bayes' Theorem states that this conditional probability can be identified as the following fraction:

$$P(A_1 \mid B) = \frac{P(B \mid A_1)P(A_1)}{P(B \mid A_1)\cdot P(A_1) + P(B \mid A_2)\cdot P(A_2) + \cdots + P(B \mid A_k)\cdot P(A_k)}$$

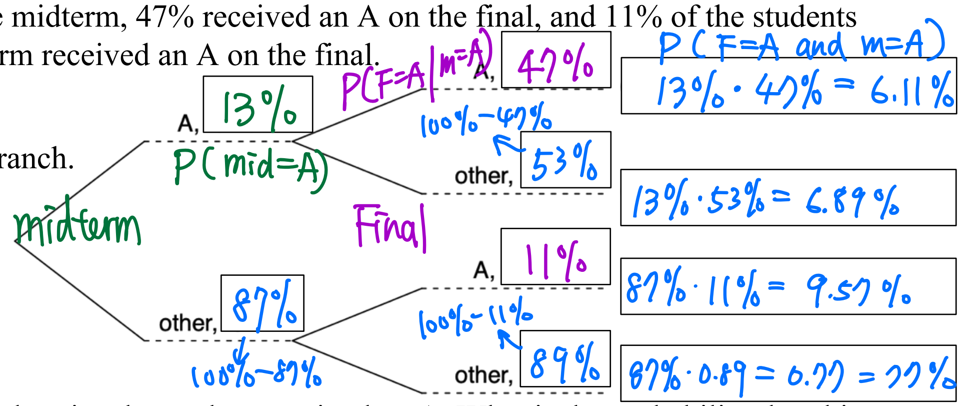
$P(B\mid A_1)P(A_1) \rightarrow P(B \text{ and } A_1)$

$\rightarrow P(B \text{ and } A_1) + P(B \text{ and } A_2) + \cdots$

where $A_2, A_3, \cdots$, and A_k represent all other possible outcomes of the first variable.

13. Consider the midterm and final for a statistics class. Suppose 13% of students earned an A on the midterm. Of those students who earned an A on the midterm, 47% received an A on the final, and 11% of the students who earned lower than an A on the midterm received an A on the final.

(a) Build a tree diagram with midterm as primary branch and final as secondary branch.



(b) You randomly pick up a final exam and notice the student received an A. What is the probability that this student earned an A on the midterm?

$$P(m=A \mid F=A) = \frac{P(F=A \text{ and } m=A)}{P(F=A \text{ and } m=A) + P(F=A \text{ and } m=\sim A)} = \frac{0.0611}{0.0611+0.0957}$$

not

$= 0.3897$