MAT1372, Classwork6, Fall2025

3.1 Defining Probability

1. PROBABILITY:

The probability of an outcome is the proportion of times the outcome would occur if we observed the random process an infinite number of times

2. A "die", the singular of dice, is a cube with six faces numbered 1, 2, 3, 4, 5, and 6. What is the chance of getting 1 when rolling a die?

If the die is fair, then the chance of 1 is as good as the chance of the other numbers.

$P(\text{rolling a } 1) = P(1) = \frac{1}{6}$

3. LAW OF LARGE NUMBERS:

As more observations are collected, the proportion $\hat{p}_n$ of occurrences with a particular outcome converges to the probability p of that outcome.

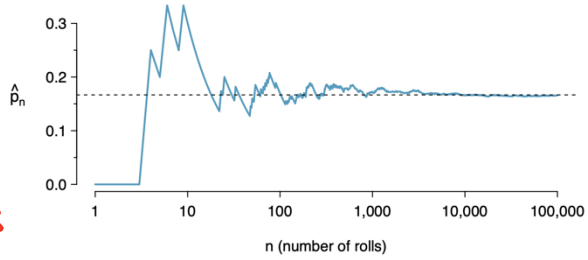


Figure 3.1: The fraction of die rolls that are 1 at each stage in a simulation. The proportion tends to get closer to the probability $1/6 \approx 0.167$ as the number of rolls increases.

4. DISJOINT:

Two outcomes are called disjoint or mutally exclusive if they cannot both happen

5. (a) What is the probability of rolling a 1, or 3? (b) What is the probability of rolling a 1, 2, 3, 4, 5, or 6?

(a) $P(1 \text{ or } 3) = P(1) + P(3) = \frac{1}{6} + \frac{1}{6} = \frac{2}{6}$

(b) $P(1, 2, 3, 4, 5, \text{ or } 6) = P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = \frac{6}{6} = 1$

6. ADDITION RULE OF DISJOINT OUTCOMES:

If A_1 and A_2 represent 2 disjoint outcomes, then the probability that one of them occurs is given by

$$P(A_1 \text{ or } A_2) = P(A_1) + P(A_2)$$

7. We are interested in the probability of rolling a 1, 4, or 5. (a) Explain why the outcomes 1, 4, and 5 are disjoint. (b) Apply the Addition Rule for disjoint outcomes to determine P (1 or 4 or 5).

(a) 1, 4, and 5 cannot happen at the same time

(b) $P(1 \text{ or } 4 \text{ or } 5) = P(1) + P(4) + P(5) = \frac{3}{6} = \frac{1}{2}$

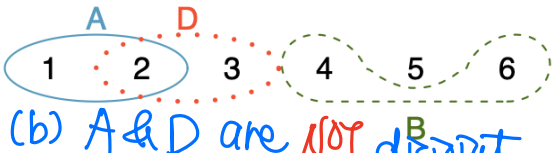
8. EVENT:

An event means a set or collection of outcome is considered. For example, if event A represents the event where a die roll results in 1 or 2, $A = \{1, 2\}$

9. (a) Are events B and D disjoint? (b) Are events A and D disjoint?

$A=\{1,2\}$ $D=\{2,3\}$

$B=\{4,6\}$ (a) Yes, B & D are disjoint



(b) A & D are NOT disjoint

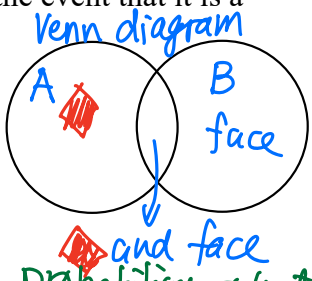
10. If you have a regular deck of 52 cards. (a) What is the probability that a randomly selected card is a diamond? (b) What is the probability that a randomly selected card is a face card?

Figure 3.3: Representations of the 52 unique cards in a deck.

(a) $P(\diamondsuit) = \frac{13}{52} = \frac{1}{4}$

(b) $P(\text{face}) = \frac{4\times 3}{52} = \frac{12}{52}$

11. Let A represent the event that a randomly selected card is a diamond and B represent the event that it is a face card. How do we compute $P(A \text{ or } B)$?



$P(\diamondsuit) + P(\text{face}) - P(\diamondsuit \text{ and face})$

$= \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52}$

12. GENERAL ADDITION RULE:

If A and B are any events (disjoint or not), then the probability that at least one of them will occur is $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

where $P(A \text{ and } B)$ means the probability that both events occur

13. If A and B are disjoint, what is $P(A \text{ and } B)$? Why?

$P(A \text{ and } B) = 0$

Because they cannot occur at the same time.

14. Independence of Random Processes

Two process are independent if knowing the outcome of one provides no useful information about the other outcome.

15. Consider rolling two dice where one is red and the other is white. What is the probability of the sum of rolling which is 2?

Red die got "1" and white die got "1"

$P(\text{sum}=2) = P(\text{Red}=1) \times P(\text{white}=1) = \frac{1}{36}$

$\frac{1}{6} \times \frac{1}{6}$

16. MULTIPLICATION RULE FOR INDEPENDENT PROCESSES

If A and B represent events from two different and independent process. Then the probability that both A and B occur can be calculated by $P(A \text{ and } B) = P(A) \times P(B)$