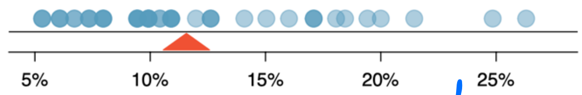
MAT1372, Classwork4, Fall2025

2.1 Examining Numerical Data

1. Dot plots and the mean



(1) Dot plots: A dot plot is a one-variable scatterplot.

(2) Mean: The mean, denoted by $\bar{x}$, is a common way to measure the center of a distribution of data.

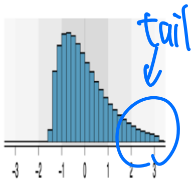
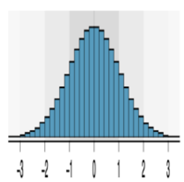
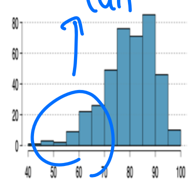
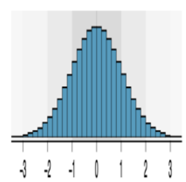
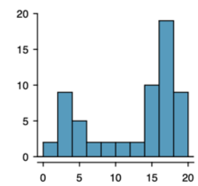
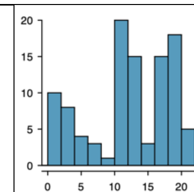
It can be computed as the sum of the observed values divided by the number of the observations:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$ where $x_1, x_2, \cdots, x_n$ represent the n observed values.

(3) Sample mean $\bar{x}$ and population mean μ: $\bar{x} \longrightarrow \mu$ (there is a natural bias b/c $\bar{x} \neq \mu$)

(4) Weighted mean: Some case's variable is more important than the same variable from other cases

2. Histograms and the shape: (1) Histogram: It provides a view of the data density.

(2) Shape			(3) Mode of a distribution		
tail 		tail 			
Right skew	symmetric	Left skew	unimodal	bimodal	multimodal

(2) Skewness: a distribution with a long tail

(3) Mode of a distribution: It is represented by the number of the prominent peaks.

3. Variance and Standard Deviation

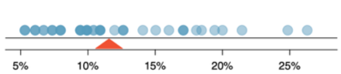
(1) Deviation	(2) Sample Variance s^2	(3) Sample standard deviation s
$x_i - \bar{x}$ for all $i = 1,2,\cdots,n$	$s^2 = \dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}$	$s = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$

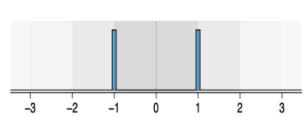
(4) Bessel's correction: $S^2_{n,\bar{x}} \le S^2_{\mu}$ population variance $S^2_{\mu} = \sum_{i=1}^{n}(x_i - \mu)^2$

$$S^2_{n,\bar{x}} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

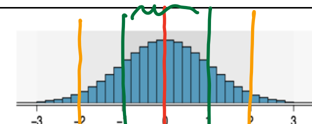
(5) Besides mean and standard variance, modality or skewness plays a role in the description of an distribution.

Why?

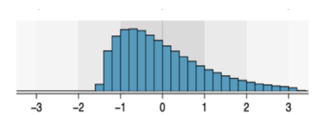




$\mu = 0$, $\sigma = 1$



, $\sigma = 1$



$\mu = 0$, $\sigma = 1$

standard deviation

4. What can the standard deviation tell us about the data?

The standard deviation represents the typical deviation of observations from the mean. Usually about 70% of the data will be with one standard deviation of mean. About 90% of the data will be with two standard deviations of mean.

5. (Bessel's correction) Given a mid-term grade for 10 students in a certain Math course:

Amy	Bert	Barry	Doug	Emily	Howard	Leo	Penny	Raj	Wil
92	95	70	95	60	30	50	70	78	80

and a sample from these 10 grades: $x = \{70, 60, 30, 50, 70\}$. Find (a) μ, (b) $\bar{x}$, (c) $s_{n,\mu}$, (d) $s_{n,\bar{x}}$, (e) s

(a)

$$\mu = \frac{92+95+70+95+60+30+50+70+78+80}{10} = \frac{720}{10} = 72$$

(b)

$$\bar{x} = \frac{70+60+30+50+70}{5} = \frac{280}{5} = 56$$

(c)

$$S_{n,\mu} = \sqrt{\frac{\sum_{i=1}^{5}(x_i-\mu)^2}{5}} = \sqrt{\frac{(70-72)^2+(60-72)^2+(30-72)^2+(50-72)^2+(70-72)^2}{5}} = \sqrt{480}$$

(d)

$$S_{n,\bar{x}} = \sqrt{\frac{\sum_{i=1}^{5}(x_i-\bar{x})^2}{5}} = \sqrt{\frac{(70-56)^2+(60-56)^2+(30-56)^2+(50-56)^2+(70-56)^2}{5}} = \sqrt{\frac{1120}{5}} = \sqrt{260}$$

(e)

$$S = \sqrt{\frac{\sum_{i=1}^{5}(x_i-\bar{x})^2}{4}} = \sqrt{\frac{1120}{4}} = \sqrt{280}$$

4 ← Bessel's correction

6. Box plots, Quartiles and the median

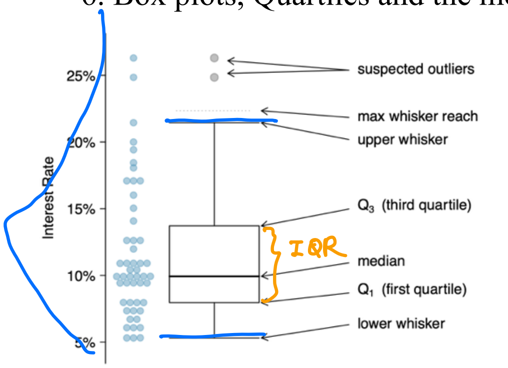


Figure 2.10: A vertical dot plot, where points have been horizontally stacked, next to a labeled box plot for the interest rates of the 50 loans.

(1) Median: If the data are ordered from smallest to the largest, the median is in the middle. If there are even number of observations, then there will be two values in the middle, and median is taken as their average

(2) The first quartile Q_1: 25% of the data fall below this value

(3) The third quartile Q_3: 75% of the data fall below this value

(4) The interquartile range IQR $= Q_3 - Q_1$: 50% of the data between Q_1 & Q_3

(5) The whiskers: upper one $Q_3 + 1.5 \times IQR$

and lower one $Q_1 - 1.5 \times IQR$

(6) Outlier: the data outside the upper / lower whiskers.

Identify skewness, possible error

7. Robust statistic:

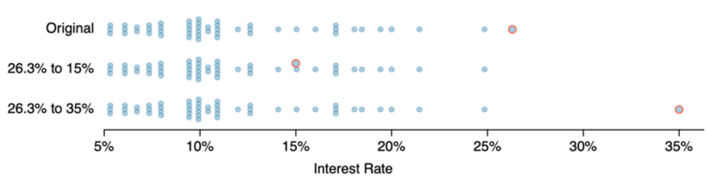


Figure 2.11: Dot plots of the original interest rate data and two modified data sets.

	robust		not robust	
scenario	median	IQR	$\bar{x}$	s
original `interest_rate` data	9.93%	5.76%	11.57%	5.05%
move 26.3% → 15%	9.93%	5.76%	11.34%	4.61%
move 26.3% → 35%	9.93%	5.76%	11.74%	5.68%

Figure 2.12: A comparison of how the median, IQR, mean ($\bar{x}$), and standard deviation (s) change had an extreme observations from the `interest_rate` variable been different.

Some statistic Variables will be changed if the extreme observations are changed. It is called robust statistics b/c extreme observations have little effect on their values.