Mat 1375 HW21

## Exercise 21.1

Write the expression as one of the six trigonometric functions.

a) $\cos(x)\cdot\tan(x)$ b) $\sec(x)\cdot\cot(x)$ c) $\frac{\csc(x)}{\sec(x)}$

a) $\cos(x)\cdot\tan(x) = \cancel{\cos(x)}\cdot\frac{\sin(x)}{\cancel{\cos(x)}} = \sin(x)$

b) $\sec(x)\cdot\cot(x) = \frac{1}{\cancel{\cos(x)}}\cdot\frac{\cancel{\cos(x)}}{\sin(x)} = \frac{1}{\sin(x)} = \csc(x)$

## Exercise 21.2

Determine if the identity is true or false. If the identity is true, then give an argument for why it is true.

a) $\cos(x)\cdot\csc(x) = \sin(x)\cdot\sec(x)$

$LHS = \cos(x)\cdot\csc(x) = \cos(x)\cdot\frac{1}{\sin(x)} = \cot(x).$

$RHS = \sin(x)\cdot\sec(x) = \sin(x)\cdot\frac{1}{\cos(x)} = \tan(x)$

$LHS \neq RHS \Rightarrow$ False.

## Exercise 21.3

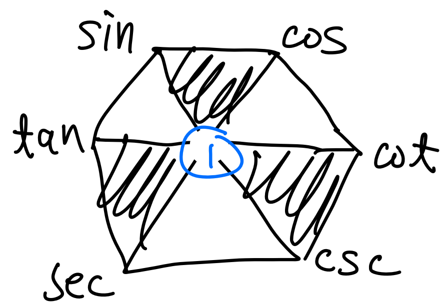
Simplify the expression as much as possible.

a) $\frac{\cos^2(x)-1}{\sin(x)}$ b) $\frac{1-\sin^2(x)}{\cot(x)}$

c) $1+\frac{\cos^2(x)}{\sin^2(x)}$ d) $\frac{\tan^2(x)}{\sec^2(x)}-1$

e) $\cos(x)+\frac{\sin^2(x)}{\cos(x)}$ f) $\sec(x)-\frac{\tan^2(x)}{\sec(x)}$

g) $(1+\sin(x))\cdot(1-\sin(x))$ h) $(1-\sec(x))\cdot(1+\sec(x))$




c) $1+\frac{\cos^2(x)}{\sin^2(x)} = \frac{\sin^2(x)+\cos^2(x)}{\sin^2(x)} \underset{\uparrow}{=} \boxed{\frac{1}{\sin^2(x)}\left(=\csc^2(x)\right)}$

$\sin^2(x)+\cos^2(x)=1$

d) $\frac{\tan^2(x)}{\sec^2(x)}-1 = \frac{\tan^2(x)-\sec^2(x)}{\sec^2(x)} \underset{\uparrow}{=} \boxed{\frac{-1}{\sec^2(x)} = -\cos^2(x)}$

$\tan^2(x)+1=\sec^2(x)$

e) $\cos(x)+\frac{\sin^2(x)}{\cos(x)} = \frac{\cos^2(x)+\sin^2(x)}{\cos(x)} = \frac{1}{\cos(x)} = \sec(x)$

f) $\sec(x) - \frac{\tan^2(x)}{\sec(x)} = \frac{\sec^2(x) - \tan^2(x)}{\sec(x)} = \frac{1}{\sec(x)} = \cos(x)$

g) $(1+\sin(x)) \cdot (1-\sin(x)) = 1 - \sin^2(x) = \cos^2(x)$

$\sin^2(x) + \cos^2(x) = 1$

## Exercise 21.4

Determine whether the identity is true or false. If the identity is true, then give an argument for why it is true.

✓a) $\sin(x) - \sin(x)\cos^2(x) = \sin^3(x)$
b) $\cot^2(x) - \csc^2(x) = \tan^2(x) - \sec^2(x)$
c) $\tan^2(x) + \sec^2(x) = 1$
d) $\sin^3(x) - \sin(x) = -\sin(x) \cdot \cos^2(x)$
✓e) $\sin(x) \cdot (\cos(x) - \sin(x)) = \cos^2(x)$
✓f) $(\sin(x) - \cos(x))^2 = 1 - 2\sin(x)\cos(x)$

$\sin^2(x) + \cos^2(x) = 1$

a) $LHS = \sin(x) - \sin(x)\cos^2(x) = \sin(x)(1 - \cos^2(x)) = \sin(x) \cdot \sin^2(x) = \sin^3(x)$

$RHS = \sin^3(x) \Rightarrow LHS = RHS$ True.

e) $\sin(x) \cdot (\cos(x) - \sin(x)) = \cos^2(x)$

$\Rightarrow \sin(x) \cdot \cos(x) - \sin^2(x) = \cos^2(x)$

$\Rightarrow \sin(x) \cdot \cos(x) = \cos^2(x) + \sin^2(x)$

$\Rightarrow \sin(x) \cdot \cos(x) = 1$ False

f) $LHS = (\sin(x) - \cos(x))^2 = \sin^2(x) + \cos^2(x) - 2\sin(x)\cos(x)$

$= 1 - 2\sin(x)\cos(x) = RHS$ True.

## Exercise 21.5

Simplify the expression as much as possible.

✓a) $\sin(x+\pi)$ ✓b) $\tan(\pi - x)$ c) $\cot(x + \frac{\pi}{2})$ d) $\cos(x + \frac{3\pi}{2})$

a) $\sin(x+\pi) = \sin(x)\underbrace{\cos(\pi)}_{=-1} + \cos(x) \cdot \underbrace{\sin(\pi)}_{=0}$

$= -\sin(x)$

b) $\tan(\pi - x) = \frac{\sin(\pi - x)}{\cos(\pi - x)} = \frac{\overset{=0}{\sin(\pi)}\cos(x) - \overset{=-1}{\cos(\pi)}\sin(x)}{\cos(\pi)\cos(x) + \sin(\pi)\sin(x)} = \frac{\sin(x)}{-\cos(x)} = -\tan(x)$

$-1$ $\qquad\qquad$ $0$

## Exercise 21.6

Find the exact values of the trigonometric functions of $\frac{\alpha}{2}$ and of $2\alpha$ by using the half-angle and double-angle formulas.

a) $\sin(\alpha) = \frac{4}{5}$, and $\alpha$ in quadrant I

b) $\cos(\alpha) = \frac{7}{13}$, and $\alpha$ in quadrant IV

c) $\sin(\alpha) = \frac{-3}{5}$, and $\alpha$ in quadrant III

b) $\cos(\alpha)=\frac{7}{13}$ and $\alpha$ in IV.

$$\sin^2(\alpha)+\cos^2(\alpha)=1 \Rightarrow \sin^2(\alpha)+\left(\frac{7}{13}\right)^2=1 \Rightarrow \sin^2(\alpha)=1-\frac{49}{169}=\frac{120}{169}$$

$$\sin(\alpha)=\cancel{\pm}-\sqrt{\frac{120}{169}}=\cancel{\pm}-\frac{2\sqrt{30}}{13} \quad (\text{since } \alpha \text{ in IV},\ \sin(\alpha)<0)$$

$$\cos(2\alpha)=\cos(\alpha)\cos(\alpha)-\sin(\alpha)\sin(\alpha)=\left(\frac{7}{13}\right)^2-\left(-\sqrt{\frac{120}{169}}\right)^2=\frac{49}{169}-\frac{120}{169}=\frac{71}{169}$$

$$\cos\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1+\cos(\alpha)}{2}}=\pm\sqrt{\frac{1+\frac{7}{13}}{2}}=\pm\sqrt{\frac{\frac{20}{13}}{2}}=\pm\sqrt{\frac{10}{13}}=\pm\frac{\sqrt{130}}{13}$$

$\alpha$ is in IV $\Rightarrow \frac{\alpha}{2}$ in II. $\Rightarrow \cos\left(\frac{\alpha}{2}\right)=-\frac{\sqrt{130}}{13}$

c) $\sin(\alpha)=-\frac{3}{5}$ and $\alpha$ in III.

$$\cos^2(\alpha)+\sin^2(\alpha)=1 \Rightarrow \cos^2(\alpha)+\left(-\frac{3}{5}\right)^2=1 \Rightarrow \cos^2(\alpha)=1-\left(\frac{9}{25}\right)=\frac{16}{25}$$

$$\Rightarrow \cos(\alpha)=\cancel{\pm}-\sqrt{\frac{16}{25}}=\cancel{\pm}-\frac{4}{5} \quad (\alpha \text{ is in III} \Rightarrow \cos(\alpha)<0)$$

$$\sin(2\alpha)=\sin(\alpha+\alpha)=\sin(\alpha)\cos(\alpha)+\cos(\alpha)\sin(\alpha)$$
$$=2\cdot\sin(\alpha)\cos(\alpha)=2\cdot\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right)=\frac{24}{25}$$

$$\sin\left(\frac{\alpha}{2}\right)=\pm\sqrt{\frac{1-\cos(\alpha)}{2}}=\pm\sqrt{\frac{1-\left(-\frac{4}{5}\right)}{2}}=\pm\sqrt{\frac{\frac{9}{5}}{2}}=\pm\sqrt{\frac{9}{10}}=\pm\frac{3}{10}\sqrt{10}$$

$\alpha$ is in III $\Rightarrow \frac{\alpha}{2}$ is in I or II $\Rightarrow \sin\left(\frac{\alpha}{2}\right)=\frac{3}{10}\sqrt{10}$

$\left(\sin\left(\frac{\alpha}{2}\right)>0\right)$