Mat 1375 HW7

Exercise 7.1

Divide by long division.

✓ a) $\frac{x^3-4x^2+2x+1}{x-2}$ ✓ b) $\frac{x^3+6x^2+7x-2}{x+3}$ ✓ c) $\frac{x^2+7x-4}{x+1}$

a)

x^2-2x-2

$x-2 \,\overline{)\, x^3-4x^2+2x+1}$

$-)\ x^3-2x^2$

$-2x^2+2x$

$-)\ -2x^2+4x$

$-2x+1$

$-)\ -2x+4$

-3

$$\frac{x^3-4x^2+2x+1}{x-2}=x^2-2x-2+\frac{-3}{x-2}$$

b)

x^2+3x-2

$x+3 \,\overline{)\, x^3+6x^2+7x-2}$

$-)\ x^3+3x^2$

$3x^2+7x$

$-)\ 3x^2+9x$

$-2x-2$

$+)\ -2x-6$

4

$$\frac{x^3+6x^2+7x-2}{x+3}=x^2+3x-2+\frac{4}{x+3}$$

(c)

$x+6$

$x+1 \,\overline{)\, x^2+7x-4}$

$-)\ x^2+x$

$6x-4$

$-)\ 6x+6$

-10

$$\frac{x^2+7x-4}{x+1}=x+6+\frac{-10}{x+1}$$

✓ j) $\frac{8x^3+18x^2+21x+18}{2x+3}$ ✓ k) $\frac{x^3+3x^2-4x-5}{x^2+2x+1}$

(j)

$4x^2+3x+6$

$2x+3 \,\overline{)\, 8x^3+18x^2+21x+18}$

$-)\ 8x^3+12x^2$

$6x^2+21x$

$-)\ 6x^2+9x$

$12x+18$

$-)\ 12x+18$

0

$$\frac{8x^3+18x^2+21x+18}{2x+3}=4x^2+3x+6$$

(k)

$x+1$

$x^2+2x+1 \,\overline{)\, x^3+3x^2-4x+5}$

$-)\ x^3+2x^2+x$

x^2-5x+5

$-)\ x^2+2x+1$

$-7x+4$

$$\frac{x^3+3x^2-4x+5}{x^2+2x+1}=x+1+\frac{-7x+4}{x^2+2x+1}$$

✓

Exercise 7.2

Find the remainder when dividing $f(x)$ by $g(x)$.

a) $f(x) = x^3 + 2x^2 + x - 3,\quad g(x) = x - 2$
b) $f(x) = x^3 - 5x + 8,\quad g(x) = x - 3$
c) $f(x) = x^5 - 1,\quad g(x) = x + 1$
d) $f(x) = x^5 + 5x^2 - 7x + 10,\quad g(x) = x + 2$

By long division, $f(x) = q(x) \cdot g(x) + r$,

If some number "c" make $g(c) = 0$, then

$f(c) = q(c) \cdot g(c) + r \Rightarrow f(c) = q(c) \cdot 0 + r = r$

Thus, we can find this kind of "c" for $g(x)$ and $f(c)$ will be the remainder

(a) $f(x) = x^3 + 2x^2 + x - 3,\quad g(x) = x - 2$

since, when $x = 2$, $g(2) = 0$, then the remainder of $\frac{f(x)}{g(x)}$

is $f(2) = 2^3 + 2 \cdot 2^2 + 2 - 3 = 8 + 8 + 2 - 3 = 15$

(b) $f(x) = x^3 - 5x + 8$. $g(x) = x - 3$

Since, when $x = 3$, $g(3) = 0$, then the remainder of $\frac{f(x)}{g(x)}$

is $f(3) = 3^3 - 5 \cdot 3 + 8 = 27 - 15 + 8 = 20$.

(c) $f(x) = x^5 - 1,\quad g(x) = x + 1$

Since, when $x = -1$, $g(-1) = 0$, then the remainder of $\frac{f(x)}{g(x)}$ is

$f(-1) = (-1)^5 - 1 = -1 - 1 = -2$

(d) $f(x) = x^5 + 5x^2 - 7x + 10,\quad g(x) = x + 2$

Since, when $x = -2$, $g(-2) = 0$, then the remainder of $\frac{f(x)}{g(x)}$ is

$f(-2) = (-2)^5 + 5 \cdot (-2)^2 - 7(-2) + 10$

$= 32 + 20 + 14 + 10 = 76$

Exercise 7.3

Determine whether the given $g(x)$ is a factor of $f(x)$. If so, name the corresponding root of $f(x)$.

a) $f(x) = x^2 + 5x + 6$, $\quad g(x) = x + 3$
b) $f(x) = x^3 - x^2 - 3x + 8$, $\quad g(x) = x - 4$
c) $f(x) = x^4 + 7x^3 + 3x^2 + 29x + 56$, $\quad g(x) = x + 7$
d) $f(x) = x^{999} + 1$, $\quad g(x) = x + 1$

a) "Check if $g(x)=x+3$ is a factor of $f(x)$" is equivalent to "check if $f(-3)=0$"
$(x+3=0 \Rightarrow x=-3)$

Then $f(-3) = (-3)^2+5(-3)+6 = 9-15+6 = 0$ implies
$g(x) = x+3$ is a factor of $f(x)$ and $x=-3$ is a root

b) "check if $g(x)=x-4$ is a factor of $f(x)$" is equivalent to "check if $f(4)=0$"
$(x-4=0 \Rightarrow x=4)$

Then $f(4) = 4^3-4^2-3\cdot 4+8 = 64-16-12+8 = 36+8 = 44 \neq 0$ implies
$g(x) = x-4$ is NOT a factor of $f(x)$

c) "Check if $g(x)=x+7$ is a factor of $f(x)$" is equivalent to "check if $f(-7)=0$"
$(x+7)=0 \Rightarrow x=-7)$

Then $f(-7) = (-7)^4+7\cdot(-7)^3+3\cdot(-7)^2+29\cdot(-7)+56$
$= 7^4 - 7^4 + 3\cdot 49 - 29\cdot 7 + 56 = -56+56 = 0$ implies
$g(x) = x+7$ is a factor of $f(x)$ and $x=-7$ is a root.

d) " Check if $g(x)=x+1$ is a factor of $f(x)$" is equivalent to "check if $f(-1)=0$"
$(x+1=0 \Rightarrow x=-1)$ → odd powers

Then $f(-1) = (-1)^{\boxed{999}}+1 = \underset{\sim}{-1}+1 = 0$ implies
$g(x) = x+1$ is a factor of $f(x)$ and $x=-1$ is a root.

Exercise 7.4

Check that the given numbers for x are roots of $f(x)$ (see Observation 7.10). If the numbers x are indeed roots, then use this information to factor $f(x)$ as much as possible.

✓a) $f(x) = x^3 - 2x^2 - x + 2,$ $\quad x = 1$
✓b) $f(x) = x^3 - 6x^2 + 11x - 6,$ $\quad x = 1, x = 2, x = 3$
✓c) $f(x) = x^3 - 3x^2 + x - 3,$ $\quad x = 3$
✓d) $f(x) = x^3 + 6x^2 + 12x + 8,$ $\quad x = -2$

a) Since $f(1) = 1^3 - 2\cdot 1^2 - 1 + 2 = 1 - 2 - 1 + 2 = 0$, then $x = 1$ is a root of $f(x)$. It implies $(x-1)$ is a factor of $f(x)$.

Thus, using long division, we have

$$\begin{array}{r|l} & x^2 - x - 2 \\ \hline x-1 & x^3 - 2x^2 - x + 2 \\ & -(x^3 - x^2) \\ \hline & \quad -x^2 - x \\ & \quad -(-x^2 + x) \\ \hline & \qquad -2x + 2 \\ & \qquad -(-2x + 2) \\ \hline & \qquad\qquad 0 \end{array}$$

$$\Rightarrow \boxed{f(x)} = (x-1)(x^2 - x - 2)$$

$$\begin{array}{cc} x & +1 \\ x & -2 \end{array}$$

$$= \boxed{(x-1)(x+1)(x-2)}$$

b) Check $f(1) = 1^3 - 6\cdot 1^2 + 11\cdot 1 - 6 = 1 - 6 + 11 - 6 = 0$ ✓ A ROOT

$f(2) = 2^3 - 6\cdot 2^2 + 11\cdot 2 - 6 = 8 - 24 + 22 - 6 = 0$ ✓ A ROOT

$f(3) = 3^3 - 6\cdot 3^2 + 11\cdot 3 - 6 = 27 - 54 + 33 - 6 = 0$ ✓ A ROOT

Since $f(x)$ is degree 3 and has exact 3 roots, then 1, 2, 3 are the three roots of $f(x)$ and $(x-1)$, $(x-2)$ $(x-3)$ are the factors of $f(x)$,

Thus
$$f(x) = 1\cdot(x-1)(x-2)(x-3)$$

c) check $f(3) = 3^3 - 3 \cdot 3^2 + 3 - 3 = 3^3 - 3^3 + 3 - 3 = 0$ ✓ A ROOT.

It implies $(x-3)$ is a factor of $f(x)$

$$
\begin{array}{r|l}
 & x^2 + 0 + 1 \\
\hline
x-3 & x^3 - 3x^2 + x - 3 \\
- & x^3 - 3x^2 \\
\hline
 & \quad 0 + x \\
- & \quad 0 + 0 \\
\hline
 & \qquad + x - 3 \\
- & \qquad x - 3 \\
\hline
 & \qquad\quad 0
\end{array}
$$

$\Rightarrow f(x) = (x-3)\underbrace{(x^2+1)}$

(can't be factorize anymore)

d) Check $f(-2) = (-2)^3 + 6(-2)^2 + 12(-2) + 8 = -8 + 24 - 24 + 8 = 0$ ✓ A ROOT

$\Rightarrow$ $(x+2)$ is a factor of $f(x)$.

$$
\begin{array}{r|l}
 & x^2 + 4x + 4 \\
\hline
x+2 & x^3 + 6x^2 + 12x + 8 \\
- & x^3 + 2x^2 \\
\hline
 & \quad 4x^2 + 12x \\
- & \quad 4x^2 + 8x \\
\hline
 & \qquad 4x + 8 \\
- & \qquad 4x + 8 \\
\hline
 & \qquad\quad 0
\end{array}
$$

$\Rightarrow f(x) = (x+2)(x^2 + 4x + 4)$

$$
\begin{array}{cc}
x & +2 \\
x & +2
\end{array}
$$

$= (x+2)(x+2)(x+2)$

Exercise 7.5

Divide by using synthetic division.

✓ a) $\frac{2x^3+3x^2-5x+7}{x-2}$ ✓ b) $\frac{4x^3+3x^2-15x+18}{x+3}$

a)

$$
\begin{array}{c|cccc}
2 & 2 & +3 & -5 & +7 \\
 & & 4 & +14 & +18 \\
\hline
 & 2 & +7 & +9 & \boxed{25}
\end{array}
$$

quotient → $2x^2 + 7x + 9$.

remainder

$2x^3 + 3x^2 - 5x + 7 = (x-2)(2x^2 + 7x + 9) + 25$

b)

$$\begin{array}{r|rrrr} -3 & 4 & +3 & -15 & +18 \\ & & -12 & +27 & -36 \\ \hline & \underbrace{4 \quad -9 \quad +12}_{} & & & \boxed{-18} \end{array}$$

quotient ↘ $4x^2-9x+12$

remainder

$$\frac{4x^3+3x^2-15x+18}{x+3} = 4x^2-9x+12+\frac{-18}{x+3}$$