Math 1375 HW2

## Exercise 2.1

For each of the following functions,

✓ a) $f(x) = 3x + 1$ ✓ b) $f(x) = x^2 - x$ c) $f(x) = \sqrt{x^2 - 9}$
d) $f(x) = \frac{1}{x}$ e) $f(x) = \frac{x-5}{x+2}$ f) $f(x) = -x^3$

calculate the function values

i) $f(3)$ ii) $f(5)$ iii) $f(-2)$ iv) $f(0)$ v) $f(\sqrt{13})$
vi) $f(\sqrt{2}+3)$ vii) $f(-x)$ viii) $f(x+2)$ ix) $f(x)+h$ x) $f(x+h)$

a) $f(x) = 3x+1$

i) $f(3) = 3\cdot 3+1 = 10$

ii) $f(5) = 3\cdot 5+1 = 16$

iii) $f(-2) = 3\cdot(-2)+1 = -6+1 = -5$

iv) $f(0) = 3\cdot 0+1 = 0+1 = 1$

v) $f(\sqrt{13}) = 3\cdot\sqrt{13}+1 = 3\sqrt{13}+1$

vi) $f(\sqrt{2}+3) = 3\cdot(\sqrt{2}+3)+1$
$= 3\sqrt{2}+9+1$
$= 3\sqrt{2}+10$

vii) $f(-x) = 3(-x)+1 = -3x+1$

viii) $f(x+2) = 3(x+2)+1 = 3x+6+1$
$= 3x+7$

ix) $f(x)+h = 3x+1+h$

x) $f(x+h) = 3(x+h)+1 = 3x+3h+1$

b) $f(x) = x^2 - x$

i) $f(3) = (3)^2-(3) = 9-3 = 6$

ii) $f(5) = (5)^2-(5) = 25-5 = 20$

iii) $f(-2) = (-2)^2-(-2) = 4+2 = 6$

iv) $f(0) = (0)^2-(0) = 0$

v) $f(\sqrt{13}) = (\sqrt{13})^2-(\sqrt{13}) = 13-\sqrt{13}$

vi) $f(\sqrt{2}+3) = (\sqrt{2}+3)^2-(\sqrt{2}+3)$

| | $\sqrt{2}$ | $+3$ |
|---|---|---|
| $\sqrt{2}$ | $2$ | $3\sqrt{2}$ |
| $+3$ | $3\sqrt{2}$ | $9$ |

$= 11+6\sqrt{2}-\sqrt{2}-3$
$= 8+5\sqrt{2}$

vii) $f(-x) = (-x)^2-(-x)$
$= x^2+x$

viii) $f(x+2) = (x+2)^2-(x+2)$

| | $x$ | $+2$ |
|---|---|---|
| $x$ | $x^2$ | $2x$ |
| $+2$ | $2x$ | $4$ |

$= x^2+4x+4-x-2$
$= x^2+3x+2.$

ix) $f(x)+h = x^2-x+h$

x) $f(x+h) = (x+h)^2-(x+h)$

| | $x$ | $+h$ |
|---|---|---|
| $x$ | $x^2$ | $xh$ |
| $+h$ | $xh$ | $h^2$ |

$= x^2+2xh+h^2-x-h$

## Exercise 2.2

Let $f$ be the piecewise defined function

$$f(x) = \begin{cases} x-5 & \text{, for } \underline{-4 < x < 3} \\ x^2 & \text{, for } \underline{3 \leq x \leq 6} \end{cases}$$

a) State the domain of the function.
Find the function values

b) $f(2)$ c) $f(5)$ d) $f(-3)$ e) $f(3)$

a) domain has all the possible input:
$\Rightarrow$ domain $= \{x \mid -4 < x < 3$ and $3 \leq x \leq 6\}$
or domain $= \{x \mid x \in (-4, 6]\}$

b) $f(2) = (2) - 5 = -3$
$-4 < 2 \leq 3 \Rightarrow$ first case

c) $f(5) = (5)^2 = 25$
$3 \leq 5 \leq 6 \Rightarrow$ the second case

d) $f(-3) = (-3) - 5 = -8$
$-4 < -3 \leq 3 \Rightarrow$ first case

e) $f(3) = (3) - 5 = -2$
$-4 < 3 \leq 3 \Rightarrow$ first case

## Exercise 2.4

Find the difference quotient $\frac{f(x+h)-f(x)}{h}$ for the following functions:

a) $f(x) = 5x$ b) $f(x) = 2x - 6$ c) $f(x) = x^2$
d) $f(x) = x^2 + 5x$ e) $f(x) = x^2 - 7$ f) $f(x) = x^2 + 3x + 4$
g) $f(x) = x^2 + 4x - 9$ h) $f(x) = 3x^2 - 2x$ i) $f(x) = 4x^2 + 6x$
j) $f(x) = 2x^2 - 8x - 3$ k) $f(x) = -5x^2 + 3$ l) $f(x) = x^3$

(b) $f(x) = 2x - 6$,
$f(x+h) = 2(x+h) - 6 = 2x + 2h - 6$
$f(x+h) - f(x) = 2x + 2h - 6 - (2x - 6)$
$= \underline{2x} + 2h \underline{-6} \underline{-2x} \underline{+6}$
$= 2h$

$\frac{f(x+h) - f(x)}{h} = \frac{2h}{h} = 2$

(f) $f(x) = x^2+3x+4$

$f(x+h) = (x+h)^2+3(x+h)+4$

$= x^2+2xh+h^2+3x+3h+4$

$f(x+h)-f(x) = x^2+2xh+h^2+3x+3h+4-(x^2+3x+4)$

$= x^2+2xh+h^2+3x+3h+4-x^2-3x-4$

$= 2xh+h^2+3h$

$\frac{f(x+h)-f(x)}{h}$

$= \frac{2xh+h^2+3h}{h}$

$= \frac{h(2x+h+3)}{h}$

$= 2x+h+3$

(g) $f(x) = x^2+4x-9$

$f(x+h) = (x+h)^2+4(x+h)-9$

$= x^2+2xh+h^2+4x+4h-9$

$f(x+h)-f(x) = x^2+2xh+h^2+4x+4h-9-(x^2+4x-9)$

$= x^2+2xh+h^2+4x+4h-9-x^2-4x+9$

$= 2xh+h^2+4h$

$\frac{f(x+h)-f(x)}{h}$

$= \frac{2xh+h^2+4h}{h}$

$= 2x+h+4.$

(h) $f(x) = 3x^2-2x$

$f(x+h) = 3(x+h)^2-2(x+h)$

$= 3(x^2+2xh+h^2)-2x-2h$

$= 3x^2+6xh+3h^2-2x-2h$

$f(x+h)-f(x) = 3x^2+6xh+3h^2-2x-2h-(3x^2-2x)$

$= 3x^2+6xh+3h^2-2x-2h-3x^2+2x$

$= 6xh+3h^2-2h$

$\frac{f(x+h)-f(x)}{h}$

$= \frac{6xh+3h^2-2h}{h}$

$= 6x+3h-2$

## Exercise 2.6

Find the domains of the following functions.

a) $f(x) = x^2 + 3x + 5$

b) $f(x) = |x - 2|$

c) $f(x) = \sqrt{x - 2}$

d) $f(x) = \sqrt{8 - 2x}$

e) $f(x) = \sqrt{|x + 3|}$

f) $f(x) = \frac{1}{x+6}$

g) $f(x) = \frac{x-5}{x-7}$

h) $f(x) = \frac{x+1}{x^2-7x+10}$

i) $f(x) = \frac{x}{|x-2|}$

j) $f(x) = \begin{cases} |x| & \text{for } 1 < x < 2 \\ 2x & \text{for } 3 \le x \end{cases}$

k) $f(x) = \frac{\sqrt{x}}{x-9}$

l) $f(x) = \frac{5}{\sqrt{x+4}}$

a) All Real numbers

c) $x \geq 2$ or $\{x \mid x \geq 2\}$ (since f(x) is not real when $x < 2$ for example, $x=1$, $f(1) = \sqrt{1-2} = \sqrt{-1}$)

d) $f(x) = \sqrt{8-2x}$, its domain is $8 - 2x \geq 0$

$\Rightarrow \frac{8}{2} \geq \frac{2x}{2} \Rightarrow 4 \geq x.$

$\Rightarrow D = \{x \mid x \leq 4\}$

f) $f(x) = \frac{1}{x+6}$. As a fraction, $x+6 \neq 0 \Rightarrow x \neq -6$

$\Rightarrow D = \{x \mid x \in \mathbb{R} \text{ but } x \neq -6\}$

g) $f(x) = \frac{x-5}{x-7}$. As a fraction, the denumerator cannot be zero

$\Rightarrow x - 7 \neq 0 \Rightarrow x \neq 7$

$\Rightarrow D = \{x \mid x \in \mathbb{R} \text{ but } x \neq 7\}$

h) $f(x) = \frac{x+1}{x^2-7x+10}$ As a fraction, the denumerator cannot be zero

which implies $x^2 - 7x + 10 \neq 0 \Rightarrow (x-2)(x-5) \neq 0$

$\begin{matrix} x & -2 \\ x & -5 \end{matrix}$

$\Rightarrow x - 2 \neq 0$ and $x - 5 \neq 0 \Rightarrow x \neq 2$ and $x \neq 5$

$\Rightarrow D = \{x \mid x \in \mathbb{R} \text{ but } x \neq 2, x \neq 5\}$