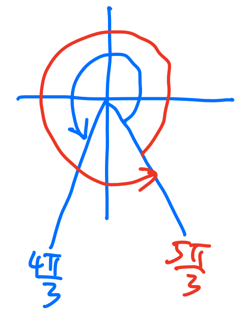
MAT1375, Classwork22, Fall2025

Ch20. Solving Trigonometric Equations

1. Find all exact solution in radians.

$2\sin^2(x) + \sqrt{3}\sin(x) = 0.$ $\Rightarrow 2\,\boxed{\sin(x)}\cdot\sin(x) + \sqrt{3}\,\boxed{\sin(x)} = 0$

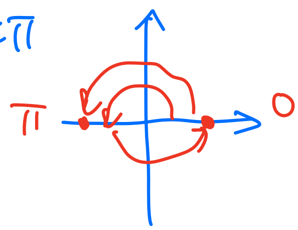
$\sin(x)\,(2\sin(x)+\sqrt{3}) = 0$



$\sin(x) = 0$

$x = \pi, 2\pi, 0, 3\pi, 4\pi$

$-\pi, -2\pi, -3\pi, -4\pi$



$\Rightarrow x = n\cdot\pi$

n: all integers

$2\sin(x)+\sqrt{3} = 0$

$\sin(x) = -\frac{\sqrt{3}}{2}$

$x = \frac{4\pi}{3}, \frac{4\pi}{3}+2\pi$ $\quad \frac{5\pi}{3}, \frac{5\pi}{3}+2\pi$

$\frac{4\pi}{3}+2\pi, \frac{4\pi}{3}-2\pi$ $\quad \frac{5\pi}{3}+4\pi, \frac{5\pi}{3}+6\pi$

$\vdots \qquad \vdots$

$\frac{4\pi}{3}+2\cdot n\pi$ $\quad \frac{5\pi}{3}+2\cdot n\cdot\pi$

n is all integer $\mathbb{Z}$ $\quad n\in\mathbb{Z}$

2. Find all exact solution in radians.

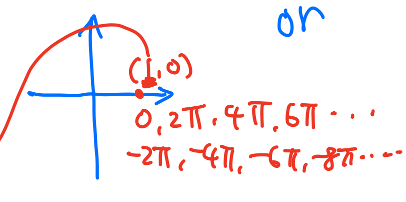
$2\cos^2(x) - \cos(x) - 1 = 0.$

$2\cos(x)\cos(x) - \cos(x) - 1 = 0$

$2\triangle^2 - \triangle - 1 = 0 \Rightarrow (\triangle-1)(2\triangle+1) = 0$ $\Rightarrow (\cos(x)-1)(2\cos(x)+1) = 0$

$\triangle \quad -1$

$2\triangle \quad +1$



$\cos(x) - 1 = 0$

$\Rightarrow \cos(x) = 1$

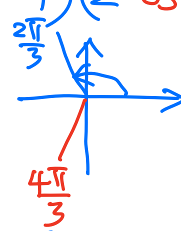
$x = 2n\pi$, $n\in\mathbb{Z}$

n is all integers

or

$2\cos(x)+1 = 0$

$\cos(x) = -\frac{1}{2}$



$x = \frac{2\pi}{3}, \frac{2\pi}{3}+2\pi, \frac{2\pi}{3}+4\pi, \frac{2\pi}{3}-2\pi \ldots$

$\Rightarrow \frac{2\pi}{3}+2n\pi$

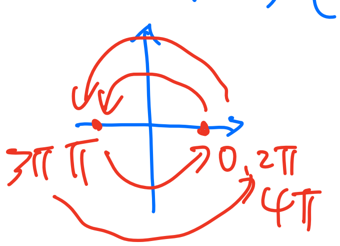
$x = \frac{4\pi}{3}+2n\pi$

3. Find all exact solution in radians.

$\tan^2(x) - \tan(x) = 0.$ $\Rightarrow \tan(x)\,(\tan(x)-1) = 0$

$\tan(x) = 0$

$\frac{\sin(x)}{\cos(x)} = 0$



$\sin(x) = 0$

$x = n\pi$

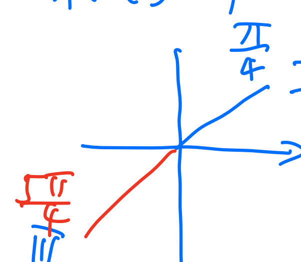
n is all integers

or

$\tan(x) - 1 = 0 \Rightarrow \tan(x) = 1$

$\frac{\sin(x)}{\cos(x)} = 1$

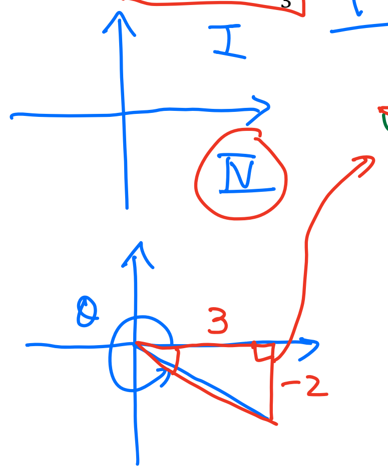
$\Rightarrow \sin(x) = \cos(x)$

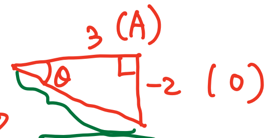


$x = \frac{\pi}{4} + n\pi$ $\quad \left(\frac{\pi}{4}+2n\pi, \frac{5\pi}{4}+2n\pi\right)$

n is all integer

4. Given $\tan(\theta) = -\frac{2}{3}$ and $\cos(\theta) > 0$. Find $\sin(\theta)$, $\cos(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$.





$\sqrt{(3)^2+(-2)^2} = \sqrt{13}$ (H)

$\sin(\theta) = \frac{O}{H} = \frac{-2}{\sqrt{13}}$

$\cos(\theta) = \frac{A}{H} = \frac{3}{\sqrt{13}}$

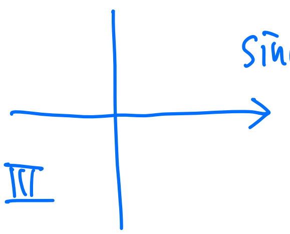
$\tan(\theta) = \frac{O}{A} = \frac{-2}{3}$

$\cot(\theta) = \frac{A}{O} = -\frac{3}{2}$

$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{\sqrt{13}}{3}$

$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{-\sqrt{13}}{2}$

5. Given $\cos(\theta) = -\frac{1}{4}$ and $\sin(\theta) < 0$. Find $\sin(\theta)$, $\tan(\theta)$, $\cot(\theta)$, $\sec(\theta)$, and $\csc(\theta)$.



Since $\cos^2(\theta) + \sin^2(\theta) = 1$

$\Rightarrow \left(-\frac{1}{4}\right)^2 + \sin^2(\theta) = 1$

$\Rightarrow \sin^2(\theta) = 1 - \frac{1}{16} = \frac{15}{16}$

$\cos(\theta) < 0$
$\sin(\theta) < 0$ $\Rightarrow$ III

$\rightarrow \sin\theta = \pm\sqrt{\frac{15}{16}} = \pm\frac{\sqrt{15}}{4}$

but $\sin\theta < 0$

$\Rightarrow \sin\theta = -\frac{\sqrt{15}}{4}$

$\sin(\theta) = -\frac{\sqrt{15}}{4}$ $\qquad \csc(\theta) = -\frac{4}{\sqrt{15}}$

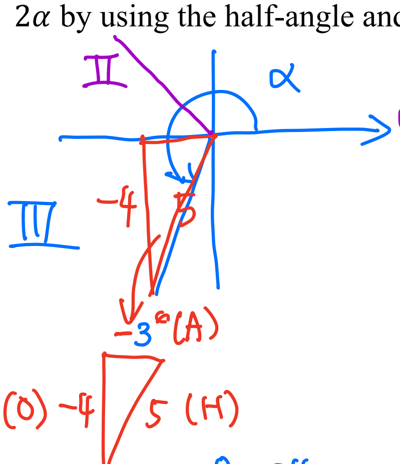
$\cos(\theta) = \frac{-1}{4}$ $\qquad \sec\theta = -4$

$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{-\frac{\sqrt{15}}{4}}{-\frac{1}{4}}$

$= \left(-\frac{\sqrt{15}}{4}\right) \times \left(\frac{-4}{1}\right) = \sqrt{15}$

$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{1}{\sqrt{15}}$

6. Given $\sin(\alpha) = -\frac{4}{5}$ and α be in quadrant III. Find the exact values of the trigonometric functions of $\frac{\alpha}{2}$ and of 2α by using the half-angle and double-angle formulas.



$\frac{\alpha}{2}$ in II

$\sin\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1-\cos(\alpha)}{2}} = +\sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}} = \sqrt{\frac{1+\frac{3}{5}}{2}} = \sqrt{\frac{\frac{8}{5}}{2}}$

$= \sqrt{\frac{8}{10}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}$

$\cos\left(\frac{\alpha}{2}\right) = \pm\sqrt{\frac{1+\cos(\alpha)}{2}}$

$\tan\left(\frac{\alpha}{2}\right)$

$\cot\left(\frac{\alpha}{2}\right)$

$\vdots$

$\sin(\alpha) = \frac{O}{H} = \frac{-4}{5}$

$\cos(\alpha) = \frac{A}{H} = \frac{-3}{5}$