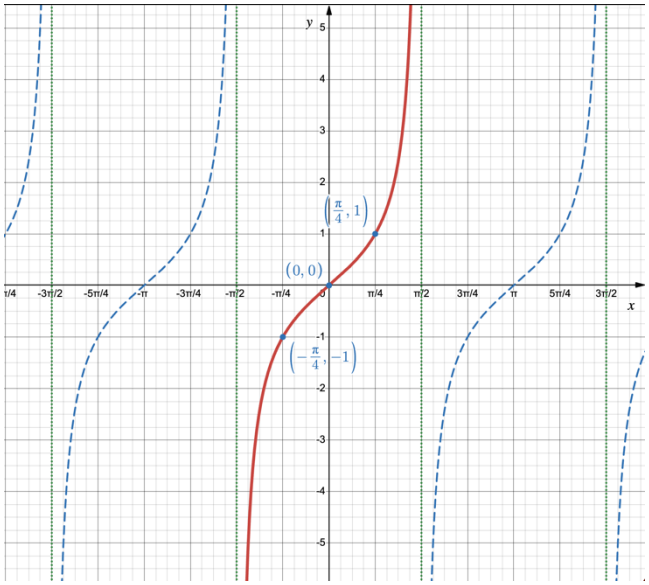
MAT1375, Classwork21, Fall2025

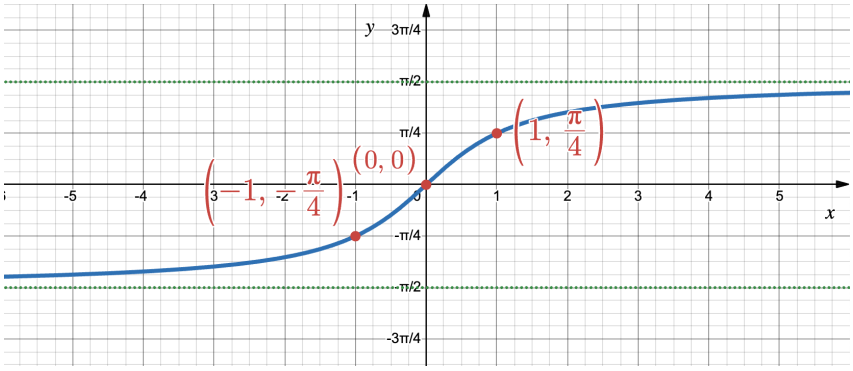
Ch19. Inverse Trigonometric Functions II

1. The graph of $y = \tan^{-1}(x)$:

$y = \tan(x)$ (the restricted tangent)



$y = \tan^{-1}(x)$



$y = \tan(x)$ (the restricted tangent)

Domain: $(-\frac{\pi}{2}, \frac{\pi}{2})$ (Angle)

Range: $(-\infty, \infty)$ (value)

V.A.: $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$

$y = \tan^{-1}(x)$

Domain: $(-\infty, \infty)$ (value)

Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$ (angle)

H.A.: $y = -\frac{\pi}{2}$, $y = \frac{\pi}{2}$

2. How to find the value of $\tan^{-1}(x)$:

Let $\theta = \tan^{-1}(x)$. It implies $x =$ $\tan(\theta)$ where $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$. Then find the θ from the following table:

θ(or $\tan^{-1}(x)$)	$-\frac{\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{4}$	$-\frac{\pi}{6}$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\tan(\theta)$(or x)	undef.	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	undefined

I

3. Find the value of the given inverse tangent functions.

a) $\tan^{-1}(\sqrt{3})$ $\frac{\pi}{3}$

d) $\tan^{-1}(-1)$ $-\frac{\pi}{4}$

4. Composition of Functions Involving with Inverse Trigonometry Functions.

a) $\sin^{-1}\left(\sin\left(\frac{\pi}{4}\right)\right)$ b) $\sin^{-1}\left(\sin\left(\frac{5\pi}{4}\right)\right)$ c) $\cos(\cos^{-1}(0.6))$ d) $\cos(\cos^{-1}(1.5))$

e) $\sin^{-1}\left(\sin\left(\frac{3\pi}{2}\right)\right)$ f) $\sin^{-1}(\sin(\pi))$ g) $\cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right)$

Sol a) $\sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$

b) $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

c) $\cos\left(\cos^{-1}(0.6)\right) = \cos(x) = 0.6$

$x = \cos^{-1}(0.6)$

$\cos(x) = 0.6$

d) $\cos\left(\cos^{-1}(1.5)\right) = \cos(x) \Rightarrow$ undefined.

$x = \cos^{-1}(1.5)$

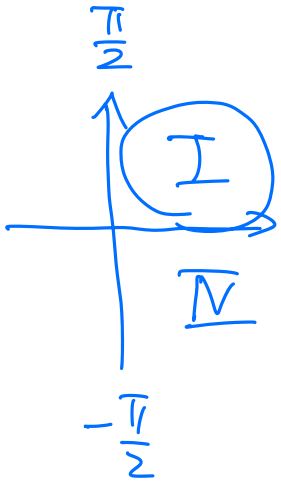
$\cos(x) = 1.5$

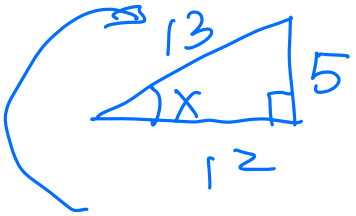
x is undefined

g) $\cos\left(\tan^{-1}\left(\frac{5}{12}\right)\right) = \cos(x) = \frac{12}{13}$ (positive)

$x = \tan^{-1}\left(\frac{5}{12}\right)$ $\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$

$\tan(x) = \frac{5}{12} > 0$

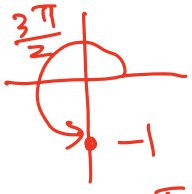




$\cos(x) = \frac{12}{13}$

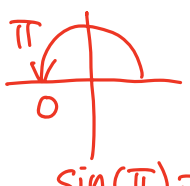
$\sqrt{5^2+12^2} = \sqrt{169} = 13$

(e) $\sin^{-1}\left(\sin\left(\frac{3\pi}{2}\right)\right) = \sin^{-1}(-1) = -\frac{\pi}{2}$



$\sin\left(\frac{3\pi}{2}\right) = -1$

(f) $\sin^{-1}\left(\sin(\pi)\right) = \sin^{-1}(0) = 0$



$\sin(\pi) = 0$