MAT1375, Classwork16, Fall2025

Ch15. Exponential Equations and Applications

1. The Exponential and Logarithmic functions and one-to-one property:

For $b > 0, b \neq 1$, the exponential and logarithmic functions are one-to-one:

Domain $(-\infty, \infty)$ $(0, \infty)$

$b^x = b^y \iff x = y$

$\log_b(x) = \log_b(y) \iff x = y$

2. Solve for x:

(a) $2^{x+7} = 32$.

$2^{x+7} = 2^5$

$x+7 = 5$

$-7 \quad -7$

$x = -2$

(b) $10^{2x-8} = 0.01$ $\frac{1}{100}$

$10^{2x-8} = \frac{1}{10^2} = 10^{-2}$

$2x-8 = -2$

$2x = 6$

$x = 3$

(c) $27^{x+3} = 9^{x-1}$.

$3^{3(x+3)} = 3^{2(x-1)}$

$\Rightarrow 3(x+3) = 2(x-1)$

$3x+9 = 2x-2$

$x = -11$

(d) $8^{x+2} = 4^{x-3}$.

$x = -12$

3. How about the equations **without** the same base on both sides? (For example, $4^x = 15$.)

$\ln(4^x) = x \cdot \ln(4)$ $\qquad \log_{10}(10^x) = x \cdot \log_{10}(10) = x$ ($\log_{10}(10) = 1$)

4. Using **Logarithms** to Solve **Exponential Equations**:

Step1: Isolate the exponential expression.

Step2: Take logarithm on both sides of the equation for common base 10 ($\log_{10}$) or natural base e ($\ln$).

Step3: Simplify using one of these properties:

$\ln(b^x) =$ $x \cdot \ln(b)$ if b is not 10 *or* $\ln(e^x) =$ $x \cdot \ln(e) = x$ (power rule, $\ln(e) = 1$) *or* $\log(10^x) =$ $x \cdot \log(10) = x \cdot 1 = x$ (power rule, $\log(10) = 1$).

Step4: Solve for x.

5. Solve for x:

$3^{x+5} = 8$.

Take "ln" on both sides

$\ln(3^{x+5}) = \ln(8)$

By power rule

$(x+5) \cdot \ln(3) = \ln(8)$

$x \cdot \ln(3) + 5 \cdot \ln(3) = \ln(8)$

$x \cdot \ln(3) = \ln(8) - 5 \cdot \ln(3)$

Divided by "ln(3)" on the both sides

$x = \dfrac{\ln(8) - 5 \cdot \ln(3)}{\ln(3)}$

6. Solve for x.

(a) $13^{2x-4} = 6$. (b) $5.1^{x} = 2.7^{2x+6}$. (c) $7e^{2x} - 5 = 58$. (d) $10^{x} = 800$.

(a) $13^{2x-4} = 6$

① Take "ln" on both sides: $\ln(13^{2x-4}) = \ln(6)$

② power rule: $(2x-4)\cdot\ln(13) = \ln(6)$

③ divided by $\ln(13)$: $2x-4 = \frac{\ln(6)}{\ln(13)}$

$\frac{2x}{2} = \frac{\frac{\ln(6)}{\ln(13)} + 4}{2}$

$x = \frac{\ln(6)}{2\ln(13)} + 2$

(c) $7e^{2x} = 63 \Rightarrow e^{2x} = 9$

$\frac{7}{7}$ $\frac{9}{7}$

$\ln(e^{2x}) = \ln(9) \Rightarrow 2x\cdot\ln(e) = \ln(9)$ ($\ln(e) = 1$)

$\Rightarrow 2x = \ln(9)$

$x = \frac{\ln(9)}{2}$

$2\cdot x\cdot\ln(13) - 4\ln(13) = \ln(6)$

$2x\cdot\ln(13) = \ln(6) + 4\ln(13)$

$x = \frac{\ln(6) + 4\ln(13)}{2\ln(13)}$

(b) $5.1^{x} = 2.7^{2x+6}$

① Take "ln" on both sides: $\ln(5.1^{x}) = \ln(2.7^{2x+6})$

② power rule: $x\ln(5.1) = (2x+6)\ln(2.7)$

$x\cdot\ln(5.1) = 2\cdot x\cdot\ln(2.7) + 6\cdot\ln(2.7)$

$x\ln(5.1) - 2x\ln(2.7) = 6\cdot\ln(2.7)$

common factor

$x\cdot(\ln(5.1) - 2\ln(2.7)) = 6\cdot\ln(2.7)$

divided

$x = \frac{6\cdot\ln(2.7)}{\ln(5.1) - 2\ln(2.7)}$

(d) $10^{x} = 800$

① Take "log" on both sides: $\log(10^{x}) = \log(800)$

② power rule: $x\cdot\log(10) = \log(800)$ ($\log(10) = 1$)

$x\cdot 1 = \log(800)$

$\Rightarrow x = \log(800)$