MAT1375, Classwork15, Fall2025

Ch14. Properties of Logarithms and Logarithmic Equations

1. Properties of Logarithms: ($\Delta = b^{\square} \Leftrightarrow \square = \log_b \Delta$) Let $X > 0, Y > 0, b > 0, b \neq 1$.

Exponential Identities	Logarithmic Identities
$b^x \cdot b^y = b^{x+y}$ (The same base multiplication = the addition of exponents)	$\log_b X + \log_b Y = \log_b(XY)$ (**The** product **Rule:** The same log base addition = the product of the values)
$\frac{b^x}{b^y} = b^{x-y}$ (The same base division = the subtraction of exponents)	$\log_b X - \log_b Y = \log_b \left(\frac{X}{Y}\right)$ (**The** Quotient **Rule:** The same log base subtraction = the division of the values)
$(b^x)^n = b^{xn}$	$\log_b X^n = n \cdot \log_b X$ (**The** power **Rule)**

$\log_b(b^x) = x$

$x \log_b(b) = x \cdot 1 = x$

The proof of Product Rule:

Let $X = b^x$, $Y = b^y$. We have $x = \log_b X$ and $y = \log_b Y$.

Then $X \cdot Y = \boldsymbol{b^x \cdot b^y = b^{x+y}} = b^{(\log_b X + \log_b Y)}$ implies

$X \cdot Y = b^{(\log_b X + \log_b Y)} \Rightarrow \log_b \Delta = \square$

$\log_b X + \log_b Y = \log_b(XY)$.

Similarly, please try to prove the quotient rule and power rule if you are interested.

2. Combine the terms using the properties of logarithms to write as one logarithm.

(a) $\frac{1}{2}\ln(x) + \ln(y)$.

(power rule) $= \ln(x)^{\frac{1}{2}} + \ln(y)$

(product rule) $= \ln(x^{\frac{1}{2}} \cdot y)$ or $\ln(\sqrt{x} \cdot y)$

(b) $5 + \log_2(a^2 - b^2) - \log_2(a+b)$ (power rule)

$5 \cdot 1 = 5 \cdot \log_2(2) = \log_2(2^5)$

$= \log_2(2^5) + \log_2(a^2 - b^2) - \log_2(a+b)$

product $= \log_2[2^5 \cdot (a^2 - b^2)] - \log_2(a+b)$

quotient $= \log_2\left[\frac{2^5 \cdot (a^2 - b^2)}{(a+b)}\right]$ or $\log_2(2^5(a-b))$

$a^2 - b^2 = (a+b)(a-b)$

each output only gets one unique input

3. The Exponential and Logarithmic functions and one-to-one property:

For $b > 0, b \neq 1$, the exponential and logarithmic functions are one-to-one:

$$\text{output} \quad b^x = b^y \quad \Leftrightarrow \quad x = y \quad \text{input}$$

$$\text{output} \quad \log_b(x) = \log_b(y) \quad \Leftrightarrow \quad x = y \quad \text{input}$$

4. Solve for x:

(a) $\log_2(x+5) = \log_2(x+3) + 4$.

$\log_2(x+5) - \log_2(x+3) = 4$

quotient rule $\Rightarrow$ $\log_2\left(\frac{x+5}{x+3}\right) = 4$

switch to exp. $\Rightarrow$ $\left(\frac{x+5}{x+3}\right) = 2^4$

$\Rightarrow$ $x+5 = 16(x+3)$

$\Rightarrow$ $x+5 = 16x+48$

$-43 = 15x$

$\Rightarrow$ $x = \frac{-43}{15}$

check

$x+5>0$ ✓

$x+3>0$ ✓

$\Downarrow$

$\boxed{x = -\frac{43}{15}}$

(b) $\log(x) + \log(x+4) = \log(5)$.

product rule

$\log(x\cdot(x+4)) = \log(5)$

$x(x+4) = 5 \Rightarrow x^2+4x-5=0$

$\Rightarrow (x-1)(x+5)=0$

$\Rightarrow x-1=0$ or $x+5=0$

$\Rightarrow \boxed{x=1}$ or $\cancel{x=-5}$

Check

$x>0$ ✓ ✗

$x+4>0$ ✓ ✗

(c) $\ln(x+2) + \ln(x-3) = \ln(7)$.

Product rule

$\ln(x+2)(x-3) = \ln(7)$

$\Rightarrow (x+2)(x-3) = 7$

$\Rightarrow x^2-x-6 = 7$

$\Rightarrow x^2-x-13=0$

using formula $x = \frac{1\pm\sqrt{1+4\cdot 13}}{2}$

$= \frac{1\pm\sqrt{53}}{2}$

$x = \frac{1+\sqrt{53}}{2} > 4$ | $x = \frac{1-\sqrt{53}}{2} < \frac{1-7}{2} = -3$

$(\sqrt{53} > \sqrt{49} = 7)$

check

$x+2>0$ ✓ | ✗

$x-3>0$ ✓ | ✗

$\Rightarrow \boxed{x = \frac{1+\sqrt{53}}{2}}$

(d) $\log_5(x-7) + \log_5(2-x) = \log_5(4)$.

product rule

$\log_5((x-7)\cdot(2-x)) = \log_5(4)$

$(x-7)(2-x) = 4$

$-x^2+9x-14=4$

$-(-x^2+9x-18=0) \Rightarrow x^2-9x+18=0$

$(x-3)(x-6)=0 \Rightarrow (x-3)=0$ or $(x-6)=0$

$\cancel{x=3}$ or $\cancel{x=6}$

check

$x-7>0$ ✗ ✗

$2-x>0$ ✗ ✗

No solution.

(x has to be in the Domain)