MAT1375, Classwork14, Fall2025

Ch13. Exponential and Logarithmic Functions II

1. Rewrite the equation in its equivalent exponential form.

a) $x = \log_2(16)$.

$2^x = 16$

$\Rightarrow x = 4$

b) $2 = \log_5 x$.

$5^2 = x$

$\Rightarrow x = 25$

c) $x = \log_{13}(1)$.

$13^x = 1$

$\Rightarrow x = 0$

d) $x = \ln(e^7) = \log_e(e^7)$

$e^x = e^7$ (since e^x is an one-to-one function)

$\Rightarrow x = 7$

2. Evaluate the expression by rewriting it as an exponential expression.

a) $\log_5(125)$.

a) $x = \log_5(125)$

$5^x = 125 \Rightarrow x = 3$

b) $\log_4(1)$.

b) $x = \log_4(1)$

$4^x = 1$

$x = 0$

c) $\log_7\left(\frac{1}{49}\right)$.

$x = \log_7\left(\frac{1}{49}\right)$

c) $7^x = \frac{1}{49}$

$7^x = 7^{-2}$

$\Rightarrow x = -2$

d) $\log_2(\sqrt[5]{2})$.

$x = \log_2(\sqrt[5]{2})$

$2^x = \sqrt[5]{2}$

$2^x = 2^{\frac{1}{5}}$

$\Rightarrow x = \frac{1}{5}$

e) $\log_{25}(5)$.

$x = \log_{25}(5)$

$25^x = 5$ * Try to find the same base on both sides

$(5^2)^x = 5$

$5^{2x} = 5^1$

$2x = 1$

$\Rightarrow x = \frac{1}{2}$

3. Graph $f(x) = 3^x$ and $g(x) = \log_3 x$ in the same coordinate.

D: $(-\infty, \infty)$

x	-3	-2	-1	0	1	2
$3^x = f(x)$	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

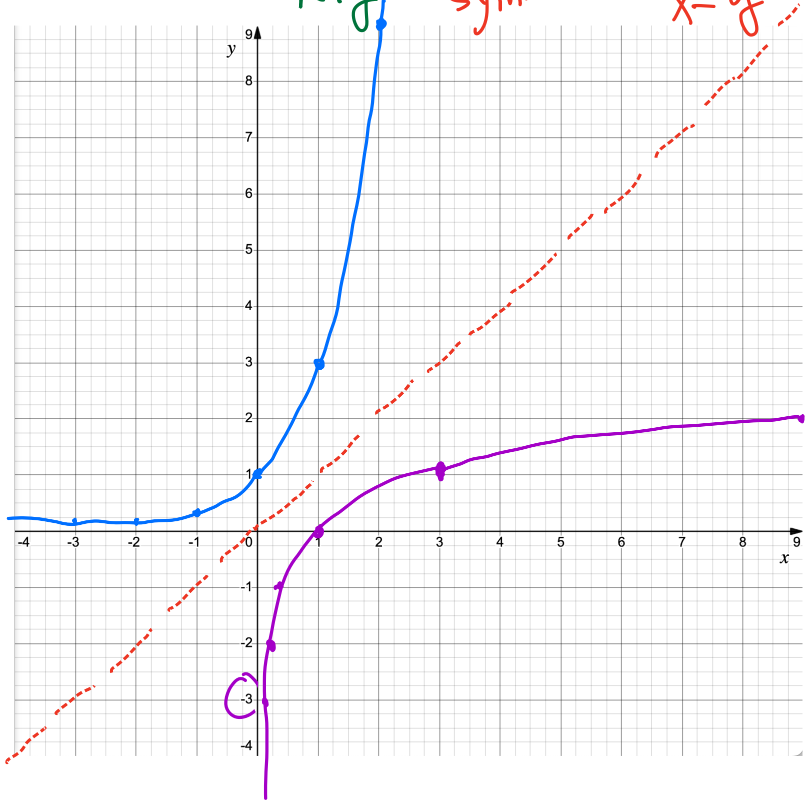
Range $(0, \infty)$

D $(0, \infty)$

x	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9
$g(x)$	-3	-2	-1	0	1	2

$= \log_3(x)$ R: $(-\infty, \infty)$

symmetric with $x = y$



$-3 = \log_3 x \Longrightarrow x = 3^{-3} = \frac{1}{27}$, $\quad 3^{-3} = x$

$-2 = \log_3 x \Longrightarrow x = 3^{-2} = \frac{1}{9}$, $\quad 3^{-2} = x$

$-1 = \log_3 x \Longrightarrow x = 3^{-1} = \frac{1}{3}$, $\quad 3^{-1} = x$

$0 = \log_3 x \Longrightarrow x = 3^0 = 1$ $\quad 3^0 = x$

$1 = \log_3 x \Longrightarrow x = 3^1 = 3$, $\quad 3^1 = x$

$2 = \log_3 x \Longrightarrow x = 3^2 = 9$. $\quad 3^2 = x$

4. Characteristics of ~~Exponential~~ log Function of $g(x) = \log_b x$.

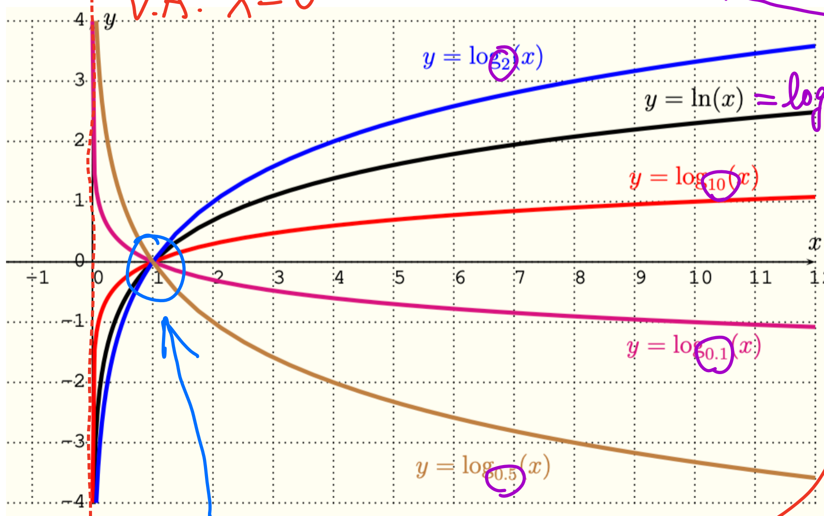
(a) The domain of g: $(0, \infty)$; The range of g: $(-\infty, \infty)$.

(b) There is NO y-intercept. In fact, g approaches, but never touches y-axis which is a vertical asymptote of g.

V.A. $X=0$

$b>1$

$e=2.71$



$0<b<1$

(c) Its x-intercept is 1 or $(1,0)$

(d) For $b > 1$, $g(x) \to \infty$ as $x \to \infty$, $g(x) \to -\infty$ as $x \to 0^+$.

(e) For $0 < b < 1$, $g(x) \to -\infty$ as $x \to \infty$, $g(x) \to \infty$ as $x \to 0^+$.

5. **Basic logarithmic evaluations**: Let $f(x) = b^x$ and $g(x) = \log_b x, b > 0, b \neq 1$. We have

(1) Elementary logarithms: $b = b^1 \Leftrightarrow 1 = \log_b(b)$. $1=\log_2(2^1)=\log_2(2)$

$1 = b^0 \Leftrightarrow 0 = \log_b(1)$.

(2) Inverse properties: $b^{g(x)} = b^{\log_b(x)} = x$. $(f(g(x)) = x)$ $\quad f(x)=b^x$, $g(x)=\log_b(x)$

$\log_b(f(x)) = \log_b(b^x) = x$. $(g(f(x)) = x)$

(3) Change-of-Base property: **10-base**: $\log_{10}(\cdot)$ $\quad \log_b(x) = \frac{\log_{10}(x)}{\log_{10}(b)} = \frac{\log(x)}{\log(b)}$. $\quad \log_3(5) = \frac{\log_{10}(5)}{\log_{10}(3)}$

natural base: $\log_e(\cdot)$ or $\ln(\ )$ $\quad \log_b(x) = \frac{\log_e(x)}{\log_e(b)} = \frac{\ln(x)}{\ln(b)}$. $\quad = \frac{\ln(5)}{\ln(3)}$

6. Given $f_1(x) = \log_e(x), f_2(x) = \log_{0.5}(x), f_3(x) = \log_{10}(x), f_4(x) = \log_2(x)$, and $f_5(x) = \log_{0.1}(x)$. Using the following numbers to find the order of these five functions from small to larger for a fixed $x > 1$:

$\ln(2) = 0.6931, \ln(10) = 2.3026, \ln(0.1) = -2.3026, \ln(0.5) = -0.6931$

$f_1(x)=\ln(x)$, $f_2(x)=\log_{\frac{1}{2}}(x)=\frac{\ln(x)}{\ln(\frac{1}{2})}=\boxed{\frac{\ln(x)}{-0.6931}}$.

$f_3(x)=\log_{10}(x)=\frac{\ln(x)}{\ln(10)}=\boxed{\frac{\ln(x)}{2.3026}}$. $f_4(x)=\log_2(x)=\frac{\ln(x)}{\ln(2)}=\boxed{\frac{\ln(x)}{0.6931}}$

$f_5(x)=\log_{\frac{1}{10}}(x)=\frac{\ln(x)}{\ln(\frac{1}{10})}=\boxed{\frac{\ln(x)}{-2.3026}}$

$$\frac{\ln(x)}{-0.6931} < \frac{\ln(x)}{-2.306} < \frac{\ln(x)}{2.306} < \ln(x) < \frac{\ln(x)}{0.6931}$$

$$f_2 < f_5 < f_3 < f_1 < f_4$$

$\log_{\frac{1}{2}}(x)$ $\log_{\frac{1}{10}}(x)$ $\log_{10}(x)$ $\ln(x)$ $\log_2(x)$