MAT1375, Classwork13, Fall2025

Ch13. Exponential and Logarithmic Functions I

1. Definition of the **Exponential Function**:

A function f is called an exponential function with base b for **any real number x** if

$$f(x) = c \cdot b^x,$$

for some real number c and positive real number b which is called the base.

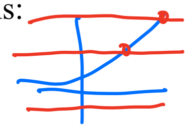
2. Please circle the given function if it is an **exponential function**:

(1) $f(x) = 2^x$. (2) $g(x) = 3^{x+1}$. (3) $h(x) = e^x$. π^x (4) $k(x) = \left(\frac{1}{5}\right)^x$. base $=\frac{1}{5}>0$ ~~(5)~~ $l(x) = x^2$. polynomial

~~(6)~~ $m(x) = (-1)^x$. base $=-1<0$ ~~(7)~~ $n(x) = x^x$.

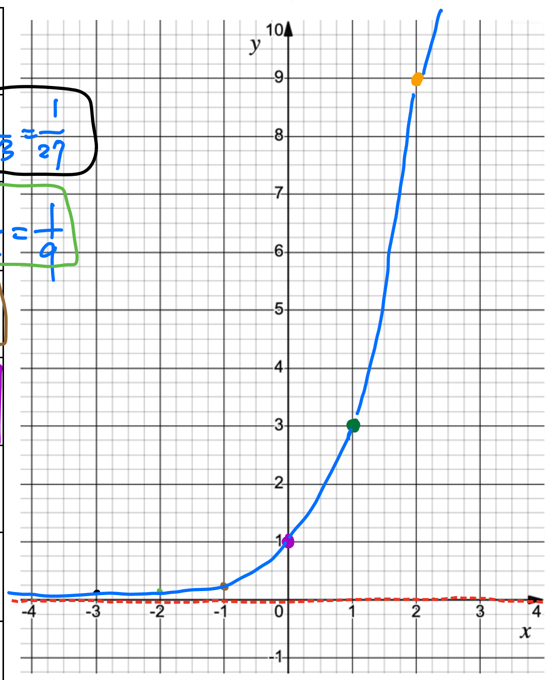
Euler's number $e = 2.71828182\cdots$
irrational number which is still a real number

3. Graph the given functions:



(a) $f(x) = 3^x$.

x	$f(x)$
-3	$3^{-3}=\frac{1}{3^3}=\frac{1}{27}$
-2	$3^{-2}=\frac{1}{3^2}=\frac{1}{9}$
-1	$3^{-1}=\frac{1}{3}$
0	$3^0=1$
1	$3^1=3$
2	$3^2=9$
3	$3^3=27$

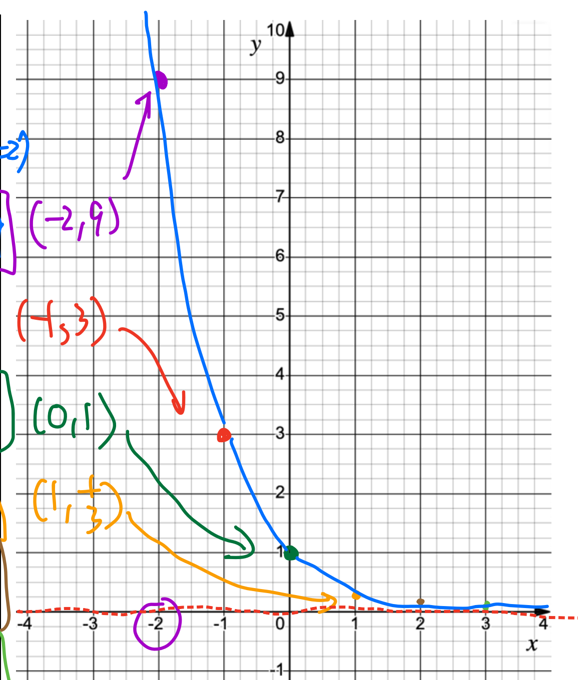


Domain: $(-\infty,\infty)$; Range: $(0,\infty)$

Asymptote: H.A. $y=0$.

(b) $g(x) = \left(\frac{1}{3}\right)^x = (3^{-1})^x = 3^{-x}$

x	$g(x)$	
-3	$3^{-(-3)}=3^3=27$	
-2	$3^{-(-2)}=3^2=9$	$(-2,9)$
-1	$3^{-(-1)}=3$	$(-1,3)$
0	$3^{-(0)}=1$	$(0,1)$
1	$3^{-(1)}=\frac{1}{3}$	$(1,\frac{1}{3})$
2	$(\frac{1}{3})^2=\frac{1}{9}$	
3	$(\frac{1}{3})^3=\frac{1}{27}$	



Domain: $(-\infty,\infty)$; Range: $(0,\infty)$.

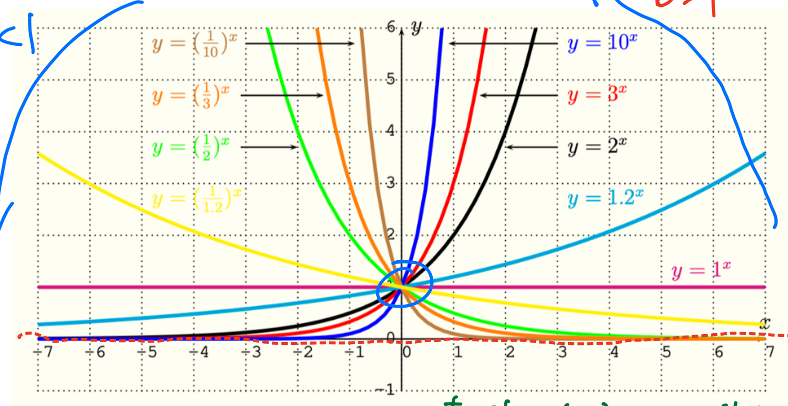
Asymptote: H.A. $y=0$.

4. Characteristics of Exponential Function of $f(x) = b^x$. $(b \neq 1)$

(a) The domain of f: $(-\infty, \infty)$; The Range of f: $(0, \infty)$.

(b) There is NO x-intercept. In fact, f approaches, but never touches x-axis which is a horizontal asymptote of f.

$0<b<1$ $\qquad$ $b>1$



HA: $y=0$

(c) Its y-intercept is 1 or $(0,1)$.

0^+: close to 0, more than 0, but $\neq 0$

(d) For $b > 1$, $f(x) \to \infty$ as $x \to \infty$, $f(x) \to 0^+$ as $x \to -\infty$.

(e) For $0 < b < 1$, $f(x) \to 0^+$ as $x \to \infty$, $f(x) \to \infty$ as $x \to -\infty$.

(f) f is one-to-one and has an inverse function.

6. What is the 4-steps strategy to find the inverse of a given function? Can it be used to find the inverse function of $f(x) = b^x$?

step 1 replace $f(x)$ by y : $y = b^x$

step 2 switch x and y : $x = b^y$

step 3 solve for y : We can not

7. Definition of **Logarithmic Function**:

For $x > 0$ and $b > 0, b \neq 1$, the logarithmic of x with base b is defined by the equivalence

$$\boxed{x = b^y \quad \Leftrightarrow \quad y = \log_b(x).}$$

This computes the inverse of the exponential function $y = b^x$ with base b. (We exchange x and y to get $x = b^y$ and solve for y).

8. Rewrite the equation as a logarithmic equation.

a) $3^4 = x$. $\qquad$ b) $e^x = 17$. $\qquad$ c) $2^{7a} = 53$. $\qquad$ d) $b^3 = 8$.

$\Leftrightarrow$

a) $4 = \log_3(x)$ $\qquad$ $\log_3(\cdot)$ $\qquad$ f

b) $x = \log_e(17)$ $\qquad$ $\ln(17)$

c) $7a = \log_2(53)$ $\Rightarrow$ $a = \frac{\log_2(53)}{7}$

d) $3 = \log_b(8)$