MAT1375, Classwork12, Fall2025

Ch12. Solving inequalities

1. The Strategy for Solving Inequalities (Application of **Number Line Test**):

Step1. Replace '>' ('≥') or '<' ('≤') by '=' and solve the equation.

Step2. Mark the solutions on the number line and check sign (positive / negative) in each subinterval.

Step3. Check the endpoint(s) of the subintervals to see if they are included in the solution set.

2. Given $x^3 + 15x \ge 7x^2 + 9$. Solve for x.

① $x^3 - 7x^2 + 15x - 9 \ge 0$.

Replace "≥" with "=", $f(x) = x^3 - 7x^2 + 15x - 9 = 0$

② Find the root(s) of $f(x) = x^3 - 7x^2 + 15x - 9$

Possible roots; 1, 3, 9, −1, −3, −9 (the factors of "−9")

$f(1) = 1^3 - 7\cdot 1^2 + 15\cdot 1 - 9 = 1 - 7 + 15 - 9 = 0 \Rightarrow x = 1$ is a root of $f(x)$

$\Rightarrow (x-1)$ is a factor of $f(x)$

$$x^2 - 6x + 9$$
$$x - 1 \,\overline{)\, x^3 - 7x^2 + 15x - 9}$$

$x^2(x-1) \rightarrow x^3 - x^2$

$-6x^2 + 15x$

$-6x(x-1) \rightarrow -6x^2 + 6x$

$9x - 9$

$9(x-1) \rightarrow 9x - 9$

0

$f(x) = (x^2 - 6x + 9)(x-1)$

$x \quad -3$

$x \quad -3$

$= (x-3)(x-3)(x-1)$

$f(x) = (x-3)(x-3)(x-1) = 0$

$x - 3 = 0$ or $x - 3 = 0$ or $x - 1 = 0$

$x = 3$ or $x = 3$ or $x = 1$.

③ Check $f(x) = x^3 - 7x^2 + 15x - 9 \ge 0$

	$-\infty$	zero 1		zero 3	∞
$f(x)$:	− (<0)		+ (>0)		+ (>0)
$(x-1)$	−		+		+
$(x-3)^2$	+		+		+

multiply

$x \in (\cancel{-\infty}, 1)$ $\quad x \in (1, 3)$ $\quad x \in (3, \infty)$

check $x = 2$ (2 is between 1 and 3)

$x \in (1, 3) \cup (3, \infty)$

④ Check end points

$x = 1$, $f(1) = 0$ included or ~~excluded~~

$x = 3$, $f(3) = 0$

$x \in [1, 3] \cup [3, \infty)$

$\boxed{\Rightarrow x \in [1, \infty)}$

3. Solve for x: $|2x-3| \geq 7$.

① $|2x-3| \geq 7$. Replace "$\geq$" with "$=$", $|2x-3| = 7$

② $2x-3=7$ or $2x-3=-7$

$x=5$ or $x=-2$

③ Check $|2x-3| \geq 7$

$x\in(-\infty,-2)$ | ~~$x\in(-2,5)$~~ | $x\in(5,\infty)$

$-\infty$ — -2 ($=7$) — 5 ($=7$) — ∞

Try $x=-3$: $|2(-3)-3| = 9 \geq 7$ True

Try $x=0$: $|2(0)-3| = 3 < 7$ False

Try $x=6$: $|2(6)-3| = 9 \geq 7$ True

$\Rightarrow x\in(-\infty,-2)\cup(5,\infty)$

④ Check end points

$x=-2$ $\quad |2\cdot(-2)-3| = 7$ included

$x=5$ $\quad |2\cdot(5)-3| = 7$ included

$$\boxed{x\in(-\infty,-2]\cup[5,\infty)}$$

4. Solve for x: $\frac{x^2-5x+6}{x^2-5x} \geq 0$.

① $f(x) = \frac{x^2-5x+6}{x^2-5x} = \frac{(x-2)(x-3)}{x(x-5)}$

$\boxed{\text{V.A. } x=0,\ x=5}$

H.A. $y=\frac{1}{1}=1$

x-intercept(s), $f(x)=0 \Rightarrow \frac{(x-2)(x-3)}{x(x-5)}=0 \Rightarrow (x-2)(x-3)=0 \Rightarrow x=2,\ x=3$

$(2,0)$ $\quad (3,0)$

y-intercept(s), $f(0)=?$ NO y-intercept (since $f(0)$ is undefined)

② $f(x)=\frac{(x-2)(x-3)}{x(x-5)}$

$x\in(-\infty,0)$ $+$ | ~~$x\in(0,2)$~~ $-$ | $x\in(2,3)$ $+$ | ~~$x\in(3,5)$~~ $-$ | $x\in(5,\infty)$ $+$

$-\infty$ — 0 (V.A.) — 2 (root) — 3 (root) — 5 (V.A.) — ∞

	$(-\infty,0)$	$(0,2)$	$(2,3)$	$(3,5)$	$(5,\infty)$
x	$-$	$+$	$+$	$+$	$+$
$x-5$	$-$	$-$	$-$	$-$	$+$
$x-2$	$-$	$-$	$+$	$+$	$+$
$x-3$	$-$	$-$	$-$	$+$	$+$

$f(x)=\frac{(x-2)(x-3)}{x(x-5)} \geq 0$

$\Rightarrow x\in(-\infty,0)\cup(2,3)\cup(5,\infty)$

③ Check endpoint(s)

Because $x=0, x=5$ are V.A., they will not be included. We only need to check $x=2, x=3$

$f(2)=\frac{(2-3)(2-2)}{2(2-5)} = \frac{-1\cdot 0}{2\cdot(-3)} = 0$ included

$f(3)=\frac{(3-3)(3-2)}{3(3-5)} = \frac{0\cdot 1}{3\cdot(-2)} = 0$ included

$$\boxed{x\in(-\infty,0)\cup[2,3]\cup(5,\infty)}$$

final answer