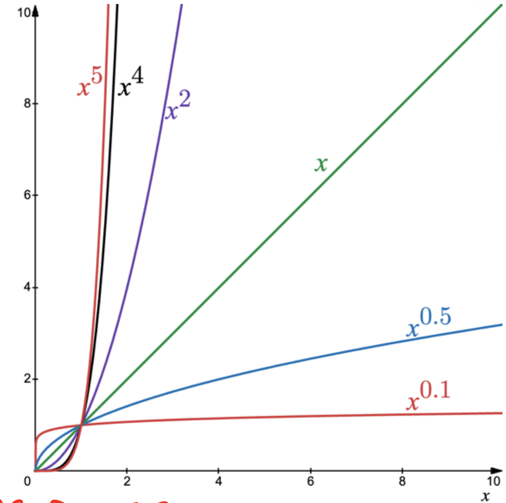
MAT1375, Classwork11, Fall2025

Ch10. Rational Functions II



6. The order of growth for functions: **Power functions**.

$$x^p, p > 0$$

For each $x > 1$, we have

$$\cdots < x^{0.1} \underline{<} x^{0.5} \underline{<} x \underline{<} x^2 \underline{<} x^4 \underline{<} x^5 < \cdots$$

7. The definition of a **Horizontal Asymptote**:

The line $\boldsymbol{y = b}$ is a horizontal asymptote of the graph of a function f

if $f(x)$ approaches $\boldsymbol{b}$ as $\boldsymbol{x}$ **increases or decreases without bound**. $x \to \infty$ or $x \to -\infty$

8. What is the difference of Vertical Asymptote and Horizontal Asymptote?

Vertical asymptote occurs at $x=c$ when $x \to c$, $f(x) \to \infty$ or $f(x) \to -\infty$ and vertical asymptote is the line $\boxed{x=c}$. On the other hand, horizontal asymptote occurs at $x \to \infty$ ($x \to -\infty$) and $f(x)$ approaches to a number c. and horizontal asymptote is the line $\boxed{y=c}$

9. How to locate Horizontal Asymptotes: Let $f(x) = \frac{p(x)}{q(x)}$ be a rational function given by

$$f(x) = \frac{p_n x^n + p_{n-1}x^{n-1} + \cdots + p_1 x + p_0}{q_m x^m + q_{m-1}x^{m-1} + \cdots + q_1 x + q_0}, \boxed{p_n \neq 0}, \boxed{q_m \neq 0.}$$

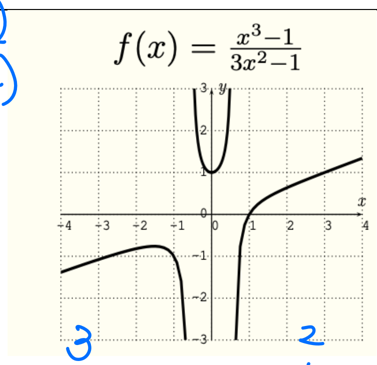
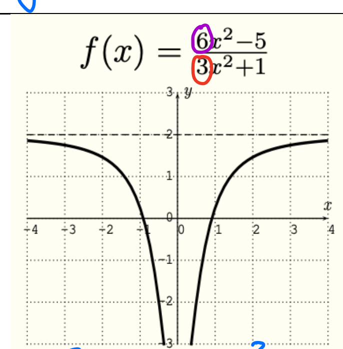
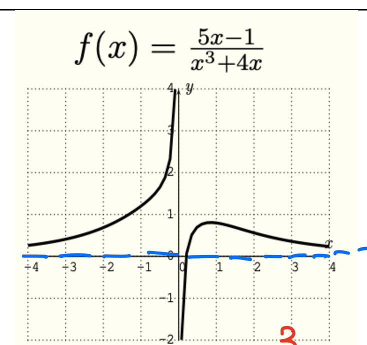
where the degree of the numerator is n and the degree of the denominator is m.

(1) If $n > m$, the graph of f has no horizontal asymptote.

(2) If $n = m$, the line $y = \frac{p_n}{q_m}$ (which is the ration of two leading coefficients) is the horizontal asymptote of the graph of f.

(3) If $n < m$, the X-axis (which is $y=0$) is the horizontal asymptote of the graph of f.

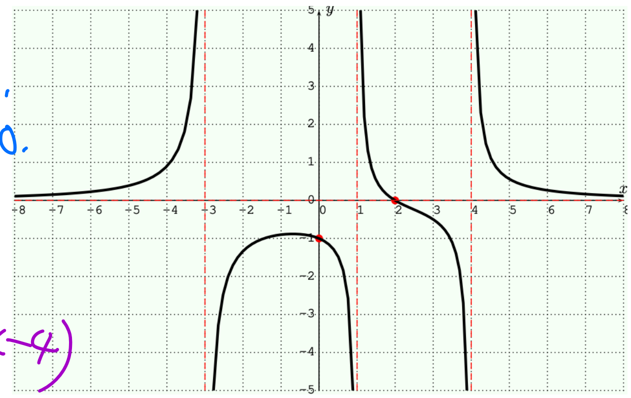
$f(x) = \frac{p(x)}{q(x)}$

 3 = deg$(p(x))$ $>$ deg$(q(x))$ = 2 no horizontal asym	 2 = deg$(p(x))$ $=$ deg$(q(x))$ = 2 $y = \frac{6}{3} = 2$	 1 = deg$(p(x))$ $<$ deg$(q(x))$ = 3 $y=0$ (x-axis)

Ch11. More on Rational Functions

1. The graph of $f(x) = \frac{p(x)}{g(x)}$ is displayed below, where $\deg(p(x)) = 1$ and $\deg(g(x)) = 3$.

Find the intercepts, asymptotes, and a formula for $f(x)$.



✓ x-intercept: $(2,0)$, $x=2$

$f(2) = \frac{p(2)}{g(2)} = 0$

$\Downarrow$

$p(2) = 0$

$(g(2) \neq 0)$

2 is a root of $p(x)$

✓ y-intercept: $(0,-1)$, $y=-1$

$\boxed{f(0)} = \frac{p(0)}{g(0)} = \boxed{-1}$

✓ V.A. $x=-3,\ x=1,\ x=4$

$(x+3)\ (x-1)\ (x-4)$

$g(x) = (x+3)(x-1)(x-4)$

✓ H.A. $y=0$.

$$f(x) = \frac{6\cdot(x-2)}{(x+3)(x-1)(x-4)} \Rightarrow f(0) = \frac{6(0-2)}{(0+3)(0-1)(0-4)}$$

$$= \frac{-2}{12}\cdot 6 = -1$$

For 2. And 3., let $f(x) = \frac{p(x)}{g(x)}$ be a rational function and $\mathbf{deg}(\boldsymbol{p}(\boldsymbol{x})) > \mathbf{deg}(\boldsymbol{g}(\boldsymbol{x}))$.

2. Rational Function and Long Division:

   If $p(x)$ (dividen) divided by $g(x)$ (divisor) can be represented with a quotient $q(x)$ and a remainder $r(x)$ where $\deg(r(x))$ <u> $<$ </u> $\deg(g(x))$, one can rewrite $f(x)$ as

$$f(x) = \frac{p(x)}{g(x)} = \underline{\ q(x)\ } + \frac{r(x)}{g(x)}.$$

3. Asymptotic Behavior with Slant Asymptote:

   Since $\deg(r(x)) < \deg(g(x))$, for large $|x|$ (which is $x \to \pm$ <u> ∞ </u>), we have

$$\frac{r(x)}{g(x)} \text{ approaches } \underline{\text{ zero }} \text{ so that } f(x) \underline{\xrightarrow{\text{approaches}}} q(x).$$

   If $q(x)$ is a <u>linear</u> function (which is a polynomial of degree <u> 1 </u>), then q is called the <u> slant </u> asymptote of f.

4. Find the **slant asymptote** of the rational function $f(x) = \frac{2x^3 - 13x^2 + 35x - 26}{x^2 - 4x + 6}$.

$$\begin{array}{r|l} & \ \ 2x - 5 \\ \hline x^2-4x+6 & 2x^3 - 13x^2 + 35x - 26 \\ & -(2x^3 - 8x^2 + 12x) \\ \hline & \ \ -5x^2 + 23x - 26 \\ & -(-5x^2 + 20x - 30) \\ \hline & \ \ 3x + 4 \end{array}$$

$\frac{2x^3}{x^2} = 2x$; $2x(x^2-4x+6)$

$\frac{-5x^2}{x^2} = -5$; $-5(x^2-4x+6)$

$$f(x) = 2x - 5 + \frac{3x+4}{x^2-4x+6}$$

$\Rightarrow y = 2x - 5$ is a slant asymptote of f