MAT1375, Classwork8, Fall2025

$2x^4+3x^5+1$

Ch8. Graphing Polynomials

1. The **End Behavior of the polynomials** and the **Leading Coefficient Test**:

$2x^4+2x+1$

As x goes to ∞ or $-\infty$, the graph of polynomial function

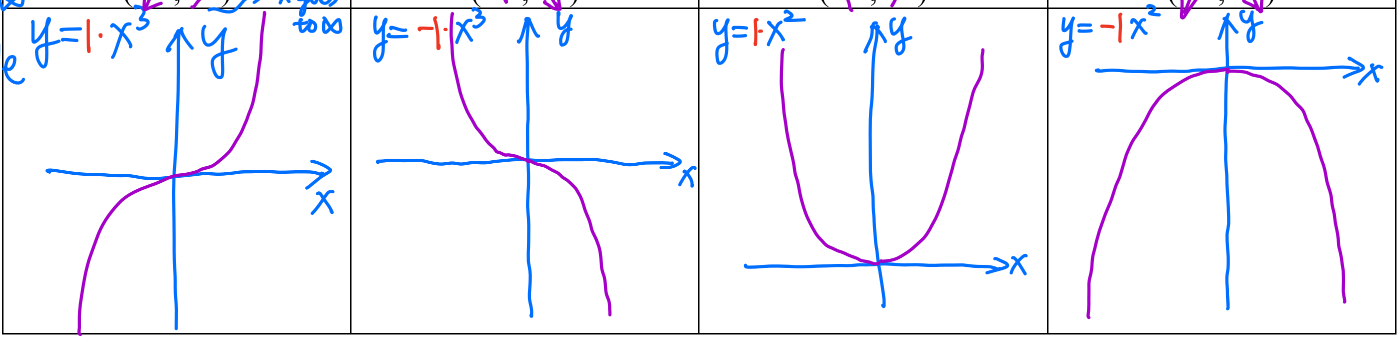
$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x^1 + a_0, \qquad (a_n \neq 0)$$

either **rises** or **falls** eventually. Here, we can conclude this into the following table

degree is odd

leading coefficient is positive

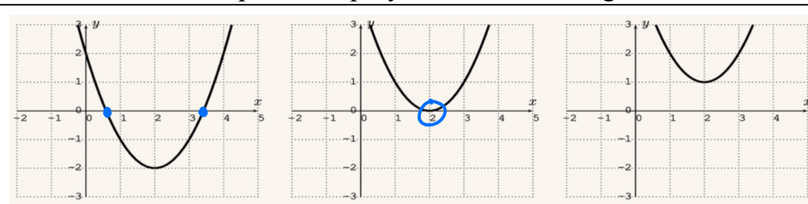
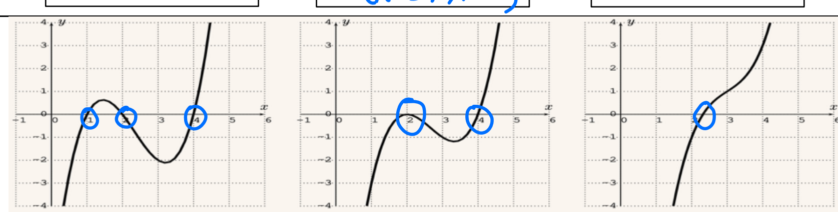
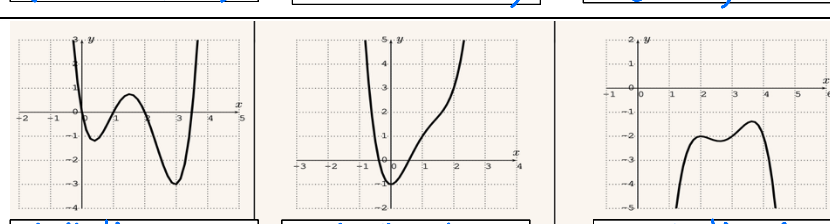
n is an **odd** number		n is an **even** number	
$a_n > 0$	$a_n < 0$	$a_n > 0$	$a_n < 0$
x goes to ∞ (↙ , ↗) x goes to ∞	(↖ , ↘)	(↖ , ↗)	(↙ , ↘)
Example $y=1\cdot x^3$	$y=-1\cdot x^3$	$y=1\cdot x^2$	$y=-1x^2$



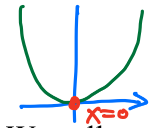
2. Roots of a Function and x-intercepts.

A root, or zero, or solution of a polynomial $f(x)$ is **a number *c*** so that $\boldsymbol{f(c)} =$ 0.

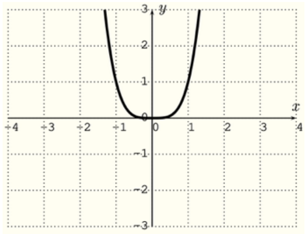
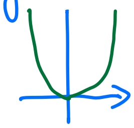
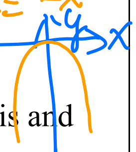
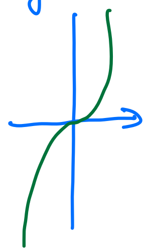
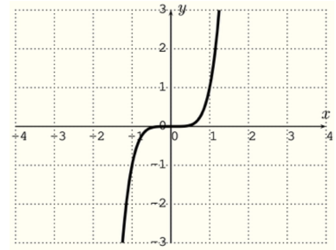
Each **real** root/zero/solution of the polynomial $f(x)$ appears as an x-intercepts of the graph of $f(x)$. (Here '**real**' means not a complex number)

Degree	Number of the roots	Graph of the polynomial of that degree
2	At most 2 (real) roots	 2 (real) roots; 1 (real) roots (x=2, x=2); No (real) roots
3	At most 3 (real) roots	 3 (real) roots (x=1, x=2, x=4); 2 (real) roots (x=2, x=2, x=4); 1 (real) root (x=2.3)
4	At most 4 (real) roots	 4 (real) roots (x=0, x=1, x=2, x=3.5); 2 (real) roots; 0 (real) root

3. **Multiplicity** of the root and x-intercepts. $y = x^2 \Rightarrow y = (x-0)^2$



Let $f(x) = (x - r)^n$ where r is the root of f and this root repeats n times. We call r a root with multiplicity n.

Even Multiplicity (n is even)	Odd Multiplicity (n is odd)
 $y = x^n$, for n even. $y=x^2$  $y = -x^2$  The graph touches the x-axis and turns around at the root r.	$y=x^3$   $y = x^n$, for n odd. The graph crosses the x-axis at the root r.
The graph tends to **flatten out** near the roots with **multiplicity** greater than 1	

4. **Turning Points** of Polynomial Functions:

Let $f(x)$ be a polynomial function of **degree n**, then the graph of f has at most $n-1$ turning points.

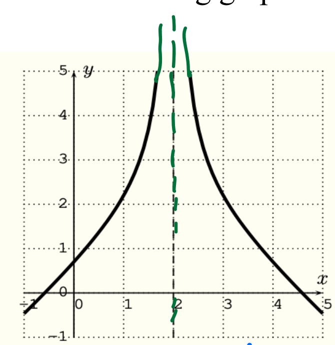
5. The essential part for drawing **a complete graph of f**:

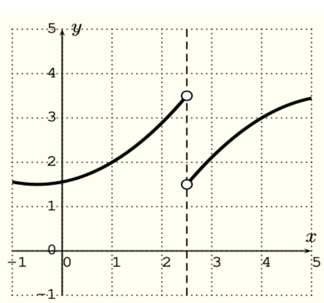
- End Behavior by Leading Coefficient test (how the function behaves when x approaches $-\infty$ / ∞)
- All roots (which are x- intercepts) with the Multiplicities
- All y-intercepts (the values by computing $f(0)$)
- All asymptotes (for rational functions in next chapter)
- Turning points with Extrema (that is all maximum and minimum)

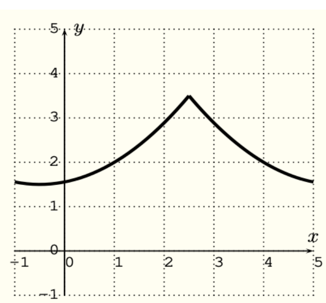
$f(x) = 3x^5 + 2x + 1$
$x = 0 \Rightarrow$ y-intercept
$f(0) = 0 + 0 + 1$
$= 1$

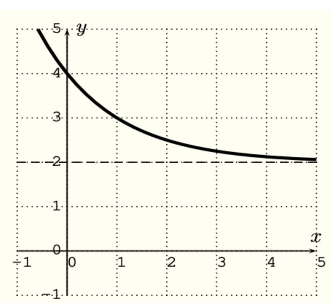
6. The domain of a polynomial f is all real numbers, and it is continuous for all real numbers and there are no jumps, no vertical or horizontal asymptotes, and no corner/cusp

The following graphs **cannot** be graphs of polynomials:


vertical asymptote


jump


corner/cusp


Horizontal asymptote