MAT1375, Classwork6, Fall2025

Ch6. The Inverse of a Function

1. (Review) Let f and g be the functions defined by the table below. Complete the table by performing the indicated operations.

x	1	2	3	4	5	6	7
$f(x)$	4	5	7	0	-2	6	4
$g(x)$	6	-8	5	2	9	11	2
$g(x)+3$	$6+3=9$	$-8+3=-5$	$5+3=8$	$2+3=5$	$9+3=12$	$11+3=14$	$2+3=5$
$f(x)-2g(x)$	$f(1)-2g(1)$ $=4-2\cdot 6=-8$	$f(2)-2g(2)$ $=5-2\cdot(-8)=21$	-3	-4	-20	-16	0
$g(x+3)$	$g(1+3)$ $=g(4)=2$	$g(2+3)$ $=g(5)=9$	$g(3+3)$ $=g(6)=11$	$g(4+3)$ $=g(7)=2$	$g(5+3)$ $=g(8)$ undefined	$g(6+3)=g(9)$ undefined	$g(7+3)=g(10)$ $=$ undefined.
$(f\circ g)(x)=f(g(x))$	$f(\boxed{g(1)})$ $=f(6)=6$	$f(g(2))$ $=f(-8)$ undefined	$f(g(3))$ $=f(5)=-2$	$f(g(4))$ $=f(2)=5$	$f(g(5))$ $=f(9)$ undefined	$f(g(6))$ $=f(11)$ undefined	$f(g(7))$ $=f(2)=5$
$(g\circ f)(x)=g(f(x))$	2	9	2	undef.	undef.	11	2
$(g\circ g)(x)=g(g(x))$	$g(g(1))$ $=g(6)=11$	$g(g(2))$ $=g(-8)$ undefined	9	-8	undefined	undefined	-8

2. Complete the definition of the **one-to-one function** (or **injective**):

Given a function $f(x)$. If any two different inputs $x_1 \neq x_2$ always have different outputs $f(x_1) \neq f(x_2)$, then we call this function f a **one-to-one function**.

3. The tables below describe assignments between inputs x and outputs y. Determine which of the given tables describe a one-to one function.

(a)

x	19	7	6	-2	3	-11
y	1	3	5	7	1	9

since input $19 \neq 3$ but they have the same output which is 1,
so, NO, this is NOT one-to-one function

(b)

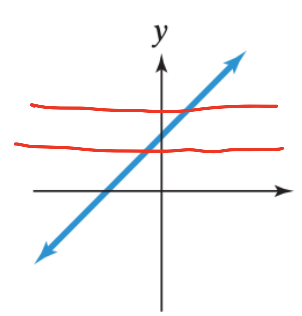
x	19	7	6	-2	3	-11
y	1	2	3	4	5	6

$19 \to 1$
$7 \to 2$
$6 \to 3$
$-2 \to 4$
$3 \to 5$
$-11 \to 6$

Yes, it is one-to-one because different inputs get different outputs

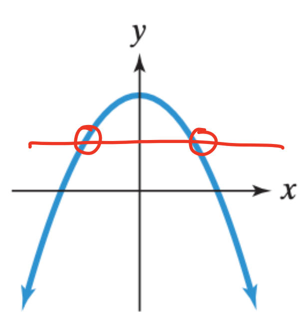
4. **Horizontal Line test**: A function is one-to-one when every horizontal line intersects the graph of the function at most once

5. Use **Horizontal Line Test** to determine which of the following are the graphs of one-to-one functions.



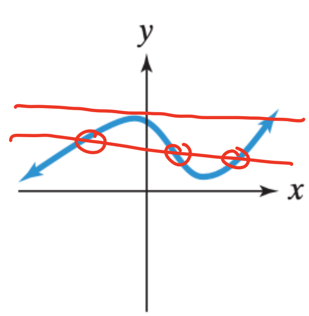
(a)

Yes, it is one-to-one



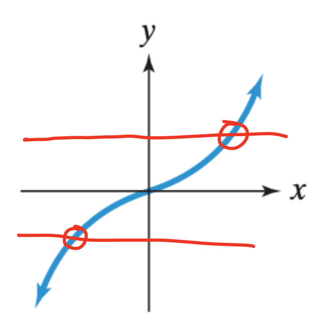
(b)

NO, it is NOT 1-to-1



(c)

NO, not 1-to-1



(d)

Yes, it is 1-to-1

6. Complete the definition of **the Inverse of a Function**:

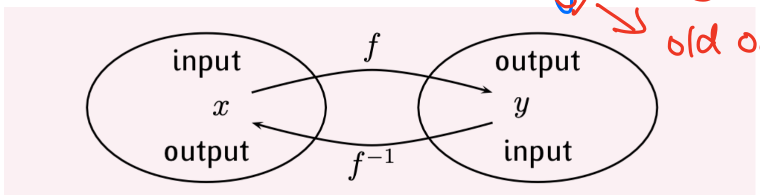
Let f be a function with domain D_f and the range R_f, and assume that f is one-to-one. The **inverse** of f is the function f^{-1}, determined by:

$\boxed{f^{-1} \neq \frac{1}{f}}$

$f(x) = y$ means precisely that $\underline{f^{-1}(y) = x}$

x → old input → new output

y → old output → new input



the domain of f^{-1}

Therefore, we have $D_{f^{-1}} = \underline{R_f}$, and $R_{f^{-1}} = \underline{D_f}$.

the range of f^{-1}

7. How to check if two given functions are **inverse** with each other:

Let f and g be two functions such that

$\underline{f(g(x)) = x}$ for every x in the domain of g **and**

$\underline{g(f(x)) = x}$ for every x in the domain of f.

The function g is the **inverse of the function *f*** and is denoted by $\underline{g = f^{-1}}$.

8. How to find the inverse function for a given **invertible** function $f(x)$:

Step1: Replace f(x) by y

Step2: Interchange x and y

Step3: solve for y (isolate y)

Step4: Replace y by $f^{-1}(x)$

$(0,1)$

input

x	0	1	2	3
$f(x)$	1	2	5	10

x^2+1

9 . Given a function $\boxed{f(x) = x^2 + 1, x \geq 0}$

(a) Find the inverse function of $f(x)$.

(b) Graph f and f^{-1} in the same coordinate system.

step1 $y = x^2+1, \ x \geq 0 \ (, \ y \geq 1)$

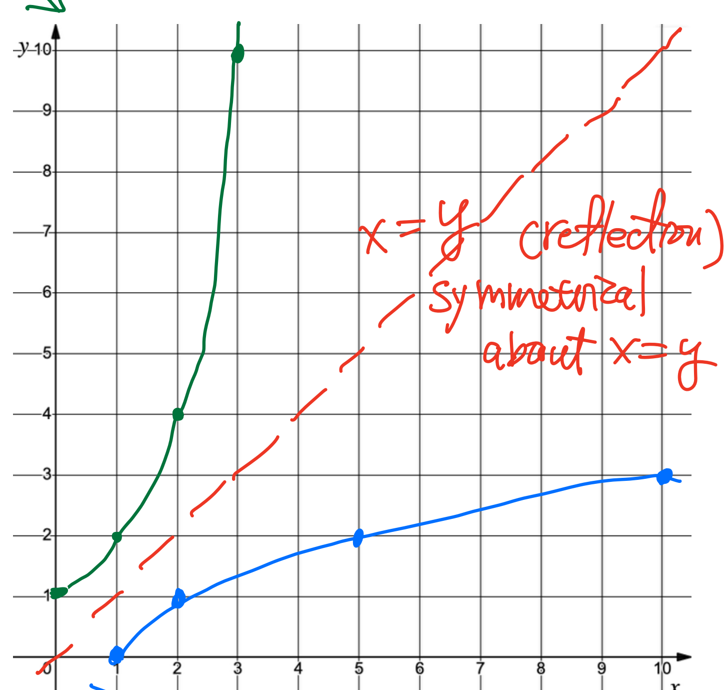
step2 $x = y^2+1, \ y \geq 0$

step3 $y^2 = x-1, \ y \geq 0$

$y = \pm\sqrt{x-1}, \ \boxed{y \geq 0}$

$\Rightarrow y = \sqrt{x-1}$.

step4 $\boxed{f^{-1}(x) = \sqrt{x-1} \quad (x \geq 1)}$



x	0	1	2	5	10
y	~~X~~	0	1	2	3

$(1,0)$ $(2,1)$ $(5,2)$

$f(x)$

input $\rightarrow$

x	0	1	2	3
$f(x)$	1	2	5	10

$f^{-1}(x)$

x	0	1	2	5	10
y	X	0	1	2	3

Given $f(x) = \dfrac{x+2}{x+1}$, $x \neq -1$. Find $f^{-1}(x)$.

step1 $\quad y = \dfrac{x+2}{x+1}$, $x \neq -1$

step2 $\quad x = \dfrac{y+2}{y+1}$, $y \neq -1$

step3

times $(y+1)$ on both sides $\quad x(y+1) = (y+2)$

$$xy + x = y + 2$$

$$xy - y = 2 - x$$

$$y(x-1) = 2 - x$$

dividing by $(x-1)$ on the both sides $\quad y = \dfrac{2-x}{x-1}$, $x \neq 1$

step4. $\quad f^{-1}(x) = \dfrac{2-x}{x-1}$, $x \neq 1$