MAT1375, Classwork5, Fall2025

Ch5. Operations on Functions

$(f-g)(x)$

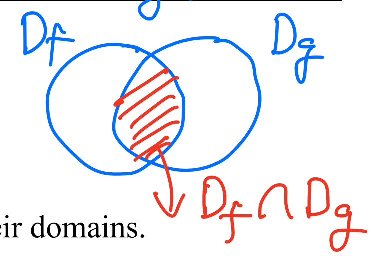
1. Complete the definition of **the Algebra of Functions**:

Let $f(x)$ and $g(x)$ be two functions with the domain D_f and D_g ,respectively. We have sum, difference, product, and quotient of functions:

intersection

The Algebra of functions	Notation	Definition	Domain
Sum	$(f+g)(x) :=$	$f(x)+g(x)$	$D_{f+g} = D_f \cap D_g$
Difference	$\boxed{(f-g)(x)} :=$	$f(x)-g(x)$	$D_{f-g} = D_f \cap D_g$
Product	$(f\cdot g)(x) :=$	$f(x)\cdot g(x)$	$D_{f\cdot g} = D_f \cap D_g$
Quotient	$\left(\frac{f}{g}\right)(x) :=$	$\frac{f(x)}{g(x)}$, provided $g(x) \neq 0$	$D_{\frac{f}{g}} = D_f \cap D_g$ but $g(x) \neq 0$

Here, $D_f \cap D_g = \{ x \mid$ x is from D_f and x is from D_g $\}$



2. Let $f(x) = x^2 + 5x + 6$ and $g(x) = x + 2$. Find the following functions and state their domains.

$(f+g)(x) = f(x)+g(x) = x^2+5x+6 + x+2$
$= x^2+6x+8$

$D_f = \mathbb{R}$ $(-\infty,\infty)$ $\quad D_g = \mathbb{R}$ $(-\infty,\infty)$

$D_{f+g} = D_f \cap D_g = \mathbb{R}$ $(-\infty,\infty)$

$(f-g)(x) = f(x)-g(x) = x^2+5x+6-(x+2)$
$= x^2+5x+6-x-2$
$= x^2+4x+4$

$D_f = \mathbb{R} \quad D_g = \mathbb{R}$

$D_{f-g} = D_f \cap D_g = \mathbb{R}$

$(f\cdot g)(x) = f(x)\cdot g(x) = (x^2+5x+6)\cdot(x+2) = x^3+7x^2+16x+12$

	x^2	$+5x$	$+6$
x	x^3	$+5x^2$	$+6x$
$+2$	$2x^2$	$+10x$	$+12$

$D_f = \mathbb{R} \quad . \quad D_g = \mathbb{R}$

$D_{fg} = D_f \cap D_g = \mathbb{R}$

$\left(\frac{f}{g}\right)(x) =$

$\frac{f(x)}{g(x)} = \frac{x^2+5x+6}{x+2} = \frac{(x+3)(x+2)}{(x+2)}$

$= (x+3)$ assuming $x+2 \neq 0$
$\Rightarrow x \neq -2$

$D_f = \mathbb{R}$, $D_g = \mathbb{R}$

$D_f \cap D_g = \mathbb{R}$ and $g(x) \neq 0$

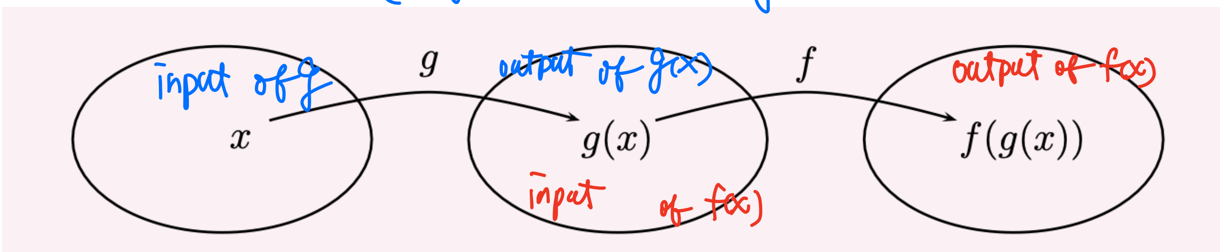
$\Rightarrow D_{\frac{f}{g}} = \{ x \mid x \in \mathbb{R}, x \neq -2 \}$

$x \in (-\infty,-2) \cup (-2,\infty)$

3. Complete the definition of **the Composition of Functions**:

Let $f(x)$ and $g(x)$ be two functions. The composition of the function f with g is denoted by $(f\circ g)(x)$ and is defined by the equation

$$(f\circ g)(x) := f(g(x)).$$



The domain of the composition of the function $f\circ g$ is the set of all x such that x is the input of $g(x)$ and output of g(x) is the domain of $f(x)$.

The notation of the domain of the composition of the function $f\circ g$ is

$D_{f\circ g}$ = { x | X is from D_g and output of g(x) is from D_f }

NO 4. Are $f(g(x))$ and $g(f(x))$ the same functions?

$f(x)$: 30% off $\Rightarrow f(x)=0.7\cdot x$

$g(x)$: 500 Gift Card $\Rightarrow g(x)=x-500$

\$3000 laptop

pay

$f(g(3000))=f(2500)$
$=2500\cdot 0.7$
$=1750$

$g(f(3000))$
$=g(3000\cdot 0.7)$
$=g(2100)$
$=2100-500$
$=1600$

5. Find $(f\circ g)(x)$ for the following functions and state their domains.

a) $f(x)=x^2+2$ and $g(x)=x-3$

$(f\circ g)(x)=f(g(x))=f(x-3)=(x-3)^2+2$

$=x^2-6x+9+2=x^2-6x+11$

	x	-3
x	x^2	$-3x$
-3	$-3x$	9

$D_{f\circ g}=(-\infty,\infty)$ or $\mathbb{R}$ or all real number

b) $f(x)=\frac{2}{x-3}$ and $g(x)=x^2+2x$

$(f\circ g)(x)=f(g(x))=f(x^2+2x)=\dfrac{2}{(x^2+2x)-3}$

$=\dfrac{2}{x^2+2x-3}$ denumerator $\neq 0$

$x^2+2x-3=(x+3)(x-1)=0$

x	3
x	-1

$\Rightarrow x+3=0$ or $x-1=0$

$\Rightarrow x=-3$ or $x=1$

$D_{f\circ g}=\{x \mid x$ is from $\mathbb{R}$ but $x\neq -3$ and $x\neq 1\}$

$=\{x \mid x\in\mathbb{R}$ but $x\neq -3$ and $x\neq 1\}$

$x\in(-\infty,-3)\cup(-3,1)\cup(1,\infty)$