MAT1375, Classwork2, Fall2025

ID:________________________________ Name:________________________________

Ch2. Functions via Formulas

1. Let $f\colon A \to B$ be a function where A is the input / domain and B is the output / codomain. If the formula of f is given, then an input $a \in A$ can find an output $b \in B$ such that $b = f(a)$.

input — belongs to

2. Given a function $f(x) = x^2 + 4x - 9$. Find the value of (a) $f(2)$; (b) $f(-3)$; (c) $f(0)$; (d) $f(h)$

input

(a) $f(2)$ $x=2$
$= (2)^2 + 4(2) - 9$
$= 4 + 8 - 9 = 3$

(b) $f(-3)$
$= (-3)^2 + 4\cdot(-3) - 9$
$= 9 - 12 - 9$
$= -12$

(c) $f(0)$
$= (0)^2 + 4(0) - 9$
$= 0 + 0 - 9$
$= -9$

(d) $f(h)$
$= (h)^2 + 4(h) - 9$
$= h^2 + 4h - 9$

3. Difference Quotient

Let $y = f(x)$ be a function. We called the expressions

$$\frac{f(x+h)-f(x)}{h} \quad \text{or} \quad \frac{f(x)-f(a)}{x-a}$$

difference quotient for the function f (which represents the slope of the secant line connecting two points on a function's graph, or the average rate of change of the function over a small interval $[x, x+h]$).

4. Given a function $f(x) = x^2 + 4x - 9$. Find the value of (a) $f(x+h)$; (b) $f(x+h) - f(x)$; (c) $\frac{f(x+h)-f(x)}{h}$.

(a) $f(x+h)$
$= (x+h)^2 + 4\cdot(x+h) - 9$
$= x^2 + 2xh + h^2 + 4x + 4h - 9$

(b) $f(x+h) - f(x)$
$= x^2 + 2xh + h^2 + 4x + 4h - 9 - (x^2 + 4x - 9)$
$= x^2 + 2xh + h^2 + 4x + 4h - 9 - x^2 - 4x + 9$
$= 2xh + h^2 + 4h$

(c) $\frac{f(x+h) - f(x)}{h}$
$= \frac{2xh + h^2 + 4h}{h}$
$= 2x + h + 4$

5. Piecewise(-Defined) Function

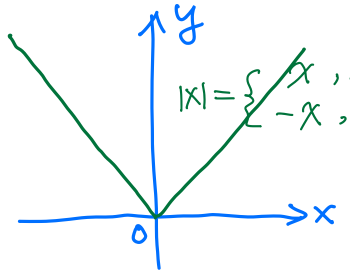
A Piecewise function is a function whose domain is partitioned into several intervals on which the function may be defined differently.

$\mathbb{R} = (-\infty, \infty)$

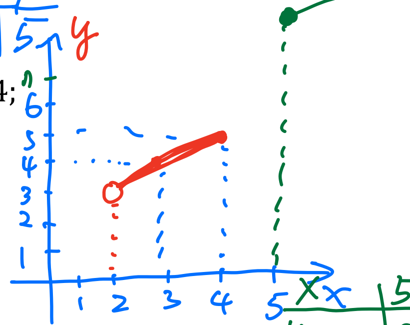
6. Examples of Piecewise Functions

x	2	3	4
$y=x+1$	3	4	5

(a) $f(x) = |x|$; (b) $f(x) = \begin{cases} x+1, & \text{for } 2 < x \le 4; \\ 2x-1, & \text{for } 5 \le x. \end{cases}$



$|x| = \begin{cases} x, & \text{for } x \ge 0 \quad [0, \infty) \\ -x, & \text{for } x < 0 \quad (-\infty, 0) \end{cases}$



x	5	6	7
$y=2x-1$	9	11	13

7. Consider the function described by the following formula. What is the domain of this function? Graph the function f.

$$f(x) = \begin{cases} x^2+1, & \text{for } -2 < x \le 0; \\ x-1, & \text{for } 0 < x \le 2; \\ -x+4, & \text{for } 2 < x \le 5. \end{cases}$$

x	-2	-1	0
y	5	2	1

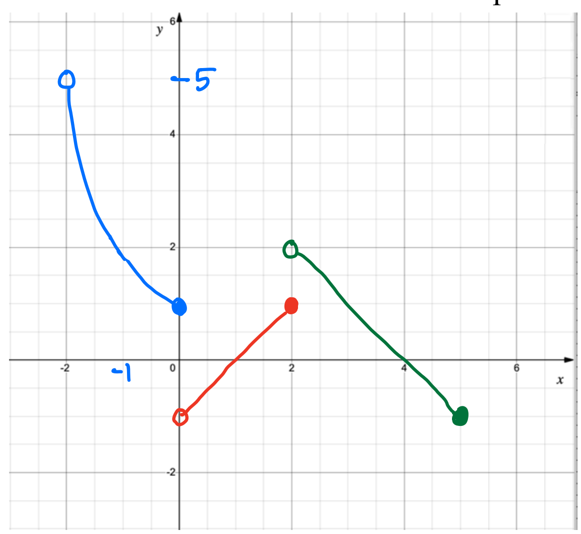
$y = x^2+1$

x	0	2
y	-1	1

$y = x-1$

x	2	5
y	2	-1

$y = -x+4$



8. Find the domain of each of the following functions according to the standard convention of the domain.

(a) $f(x) = x^2 + 4x - 9$; (b) $f(x) = |x|$; (c) $f(x) = \sqrt{x}$; (d) $f(x) = \sqrt{x-3}$; (e) $f(x) = \frac{3x+5}{x-10}$.

(a) Since, for all input in $\mathbb{R}$ have an output in $\mathbb{R}$. Then, the domain of $f(x) = x^2+4x-9$ is → All Real number → $\mathbb{R}$

(b) Since all input in $\mathbb{R}$ have an output in $\mathbb{R}$. Then, the domain of $|x|$ is $\mathbb{R}$

(c) for $f(x) = \sqrt{x}$

Since $x < 0$ as an input will NOT have a real output. Then the domain of $f(x) = \sqrt{x}$ is $\{x \mid x \ge 0\}$

(d) $f(x) = \sqrt{x-3}$

Since for $x < 3$ as an input, it doesn't have a real output. Then the domain of $f(x) = \sqrt{x-3}$ is $\{x \mid x \ge 3\}$

(e) $f(x) = \dfrac{3x+5}{x-10}$

Since if $x = 10$, $x - 10 = 0$, $f(10)$ is undefined (∞). Then, the domain of f is $\{x \mid x \in \mathbb{R} \text{ and } x \ne 10\}$

Domain: $x \in (-\infty, 10) \cup (10, \infty)$

belongs to — union