Quiz13, MAT 1375 Professor Chiu

ID:_____ Name:_____

- This quiz consists of 2 sets of questions for a total of 10 points.
- You have 15 minutes to complete the quiz.
- Scientific calculators are allowed.
- Wishing you success.

True or False questions:

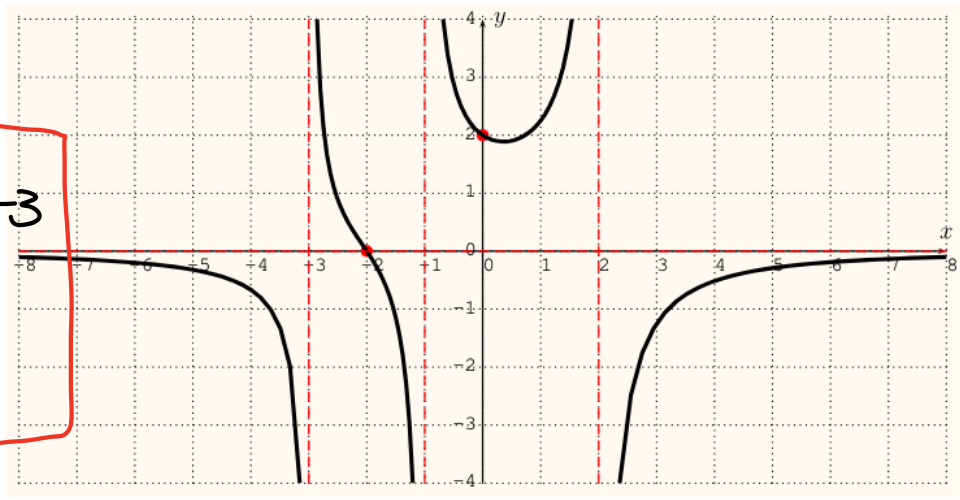
1. T Let $f(x) = \frac{(x-2)(x+3)}{x-3}$. Then $x = 3$ is a vertical asymptote of $f(x)$.
2. T Let $f(x) = \frac{3x-2}{x-3}$. Then $y = 3$ is a horizontal asymptote of $f(x)$.
3. T Let $f(x) = \frac{x^2-2}{x-3}$. Then there is no horizontal asymptote of $f(x)$.
4. T Let $f(x) = x - 3 + \frac{9}{x+3}$. Then $y = x - 3$ is a slant asymptote of $f(x)$.
5. F Let $f(x) = x^2 + x + 1 + \frac{1}{x-1}$. Then $y = x^2 + x + 1$ is a slant asymptote of $f(x)$.

↳ this is a deg 2, not deg 1, polynomial.

Please turn over for the next question.

Show all your work and justify your answer:

6. The graph of $f(x) = \frac{p(x)}{q(x)}$ is displayed below, where $\deg(p(x)) = 1$ and $\deg(q(x)) = 3$. All intercepts and asymptotes are at integer values. Find all intercepts, asymptotes, and a formula for $f(x)$.



① H.A. $y = 0$

② V.A. $x = 2,\ x = -1,\ x = -3$

③ x-intercept $(-2, 0)$

④ y-intercept $(0, 2)$

By ②, we know the zeros for $q(x)$ are $x = 2,\ x = -1,\ x = -3$

$\Rightarrow$ the factors of $q(x)$ are $(x-2),\ (x+1),\ (x+3)$

By ③, we have $f(-2) = 0 \Rightarrow f(-2) = \frac{p(-2)}{q(-2)} = 0$ $(q(-2) \neq 0)$

$\Rightarrow$ $p(-2) = 0$ which means "-2 is a zero of $p(x)$"

$\Rightarrow$ $(x+2)$ is a factor of $p(x)$

We have $p(x) = C(x+2)$ where C is a constant

Then, we have $f(x) = \frac{p(x)}{q(x)} = \frac{C(x+2)}{(x-2)(x+1)(x+3)}$

By ④, we have $2 = f(0) = \frac{p(0)}{q(0)} = \frac{C(0+2)}{(0-2)(0+1)(0+3)} = \frac{2 \cdot C}{(-2) \cdot 1 \cdot 3} = \frac{2C}{-6}$

$\Rightarrow 2 = \frac{2C}{-6} \Rightarrow 2C = -12, \Rightarrow C = -6$

Thus $f(x) = \frac{-6(x+2)}{(x-2)(x+1)(x+3)}$