For example, when multiplying $(x-(2+3i))(x-(2-3i))$, we can group the x and 2, and then use the binomial formula $(a+b)(a-b)=a^2-b^2$ to evaluate:

$$\begin{aligned}(x-(2+3i))(x-(2-3i)) &= ((x-2)-3i)((x-2)+3i)\\ &= (x-2)^2-9i^2=(x-2)^2+9\end{aligned}$$

9.3 Exercises

Exercise 9.1

a) Find all rational roots of $f(x)=2x^3-3x^2-3x+2$.
b) Find all rational roots of $f(x)=3x^3-x^2+15x-5$.
c) Find all rational roots of $f(x)=6x^3+7x^2-11x-12$.
d) Find all real roots of $f(x)=6x^4+25x^3+8x^2-7x-2$.
e) Find all real roots of $f(x)=4x^3+9x^2+26x+6$.

Exercise 9.2

Find a root of the polynomial by guessing possible candidates of the root.

a) $f(x)=x^5-1$	b) $f(x)=x^4-1$	c) $f(x)=x^3-27$
d) $f(x)=x^3+1000$	e) $f(x)=x^4-81$	f) $f(x)=x^3-125$
g) $f(x)=x^5+32$	h) $f(x)=x^{777}-1$	i) $f(x)=x^2+64$

Exercise 9.3

Find the roots of the polynomial and use it to factor the polynomial completely.

a) $f(x)=x^3-7x+6$	b) $f(x)=x^3-x^2-16x-20$
c) $f(x)=x^3-7x^2+17x-20$	d) $f(x)=x^3+x^2-5x-2$
e) $f(x)=2x^3+x^2-7x-6$	f) $f(x)=12x^3+49x^2-2x-24$
g) $f(x)=x^3-3x^2+9x+13$	h) $f(x)=x^4-5x^2+4$
i) $f(x)=x^4-1$	j) $f(x)=x^5-6x^4+8x^3+6x^2-9x$
k) $f(x)=x^3-27$	l) $f(x)=x^4+2x^2-15$

Exercise 9.4

Find the exact roots of the polynomial; write the roots in simplest radical form, if necessary. Sketch a graph of the polynomial with all roots clearly marked.

a) $f(x) = x^3 - 2x^2 - 5x + 6$
b) $f(x) = x^3 + 5x^2 + 3x - 4$
c) $f(x) = -x^3 + 5x^2 + 7x - 35$
d) $f(x) = x^3 + 7x^2 + 13x + 7$
e) $f(x) = 2x^3 - 8x^2 - 18x - 36$
f) $f(x) = x^4 - 4x^2 + 3$
g) $f(x) = -x^4 + x^3 + 24x^2 - 4x - 80$
h) $f(x) = 7x^3 - 11x^2 - 10x + 8$
i) $f(x) = -15x^3 + 41x^2 + 15x - 9$
j) $f(x) = x^4 - 6x^3 + 6x^2 + 4x$

Exercise 9.5

Find a real number C so that the polynomial has a root as indicated. Then, for this choice of C, find all remaining roots of the polynomial.

a) $f(x) = x^3 + 6x^2 + 5x + C$ has root at $x = 1$
b) $f(x) = x^3 - 4x^2 - 2x + C$ has root at $x = -2$
c) $f(x) = x^3 - x^2 - 9x + C$ has root at $x = 3$
d) $f(x) = x^3 + 8x^2 + 5x + C$ has root at $x = -1$
e) $f(x) = x^3 - 5x^2 + 15x + C$ has root at $x = 2$

Exercise 9.6

Find a polynomial f that fits the given data.

a) f has degree 3. The roots of f are precisely 2, 3, 4. The leading coefficient of f is 2.

b) f has degree 4. The roots of f are precisely -1, 2, 0, -3. The leading coefficient of f is -1.

c) f has degree 3. f has roots -2, -1, 2, and $f(0) = 10$.

d) f has degree 4. f has roots 0, 2, -1, -4, and $f(1) = 20$.

e) f has degree 3. The coefficients of f are all real. The roots of f are precisely $2 + 5i$, $2 - 5i$, 7. The leading coefficient of f is 3.

f) f has degree 3. The coefficients of f are all real. f has roots i, 3, and $f(0) = 6$.

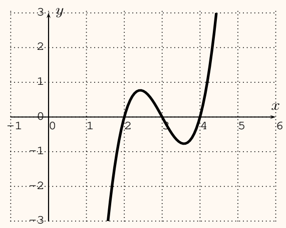
g) f has degree 4. The coefficients of f are all real. f has roots $5+i$ and $5-i$ of multiplicity 1, the root 3 of multiplicity 2, and $f(5)=7$.

h) f has degree 4. The coefficients of f are all real. f has roots i and $3+2i$.

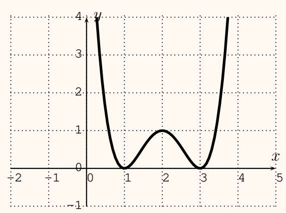
i) f has degree 6. f has complex coefficients. f has roots $1+i$, $2+i$, $4-3i$ of multiplicity 1 and the root -2 of multiplicity 3.

j) f has degree 5. f has complex coefficients. f has roots i, 3, -7 (and possibly other roots).

k) f has degree 3. The roots of f are determined by its graph:



l) f has degree 4. The coefficients of f are all real. The leading coefficient of f is 1. The roots of f are determined by its graph:



m) f has degree 4. The coefficients of f are all real. f has the following graph: