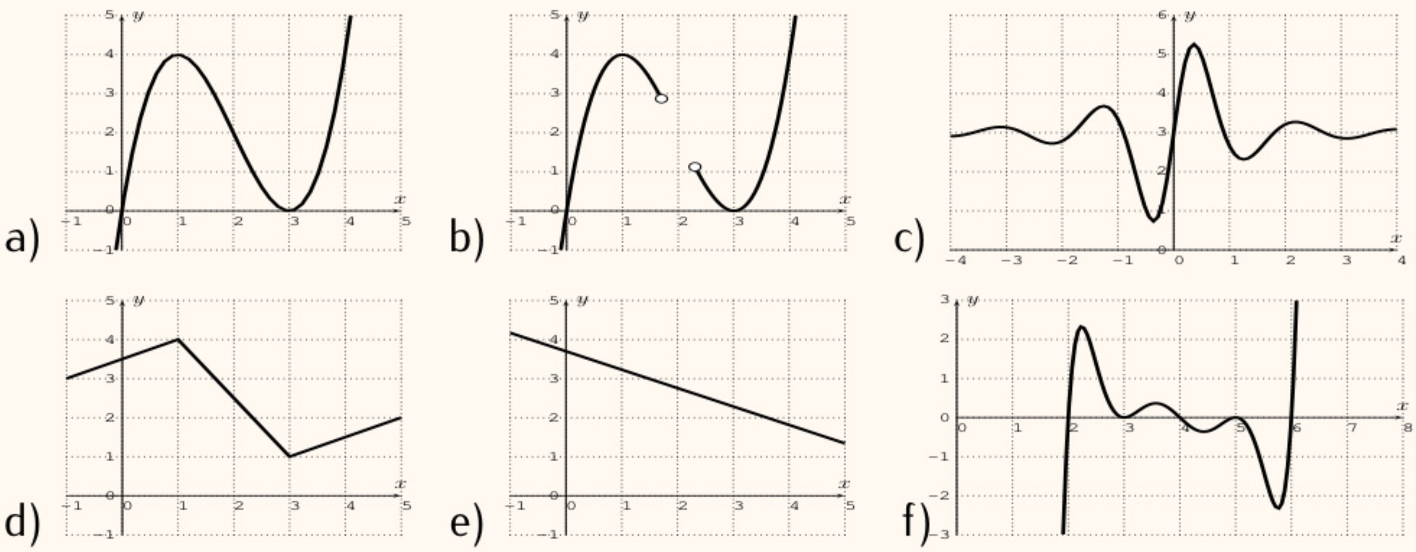
Mat 1375 HW8

Exercise 8.1

Assuming the graphs below are complete graphs, which of the graphs could be the graphs of a polynomial?



<u>Sol</u> (a) Polynomial (b) Not a polynomial (discontinuous)

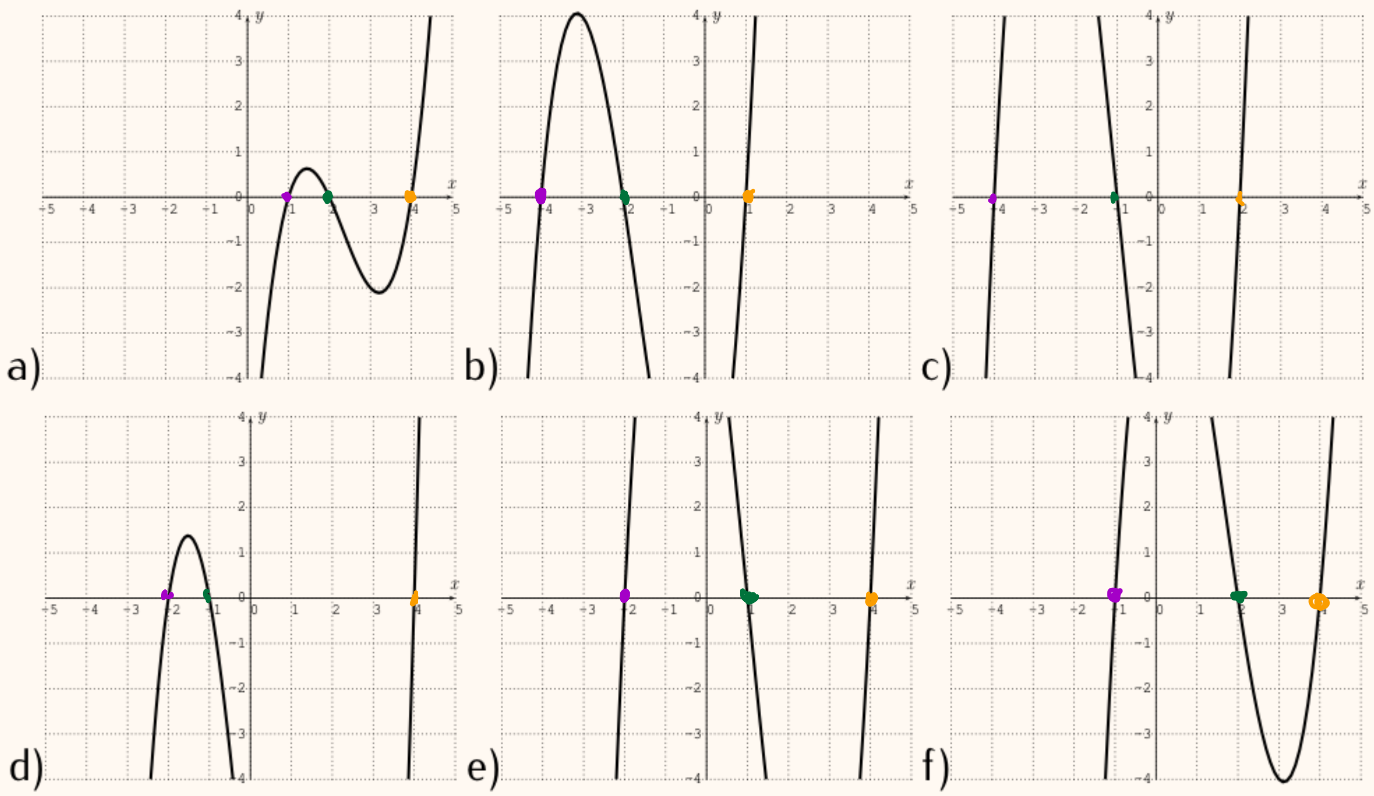
(c) Polynomial (d) Not a Polynomial (Not smooth)

(e) Polynomial (f) Polynomial

Exercise 8.2

For each of the polynomials f, g, h, and k, find the corresponding graph from (a)–(f) below.

$$f(x) = (x-1)\cdot(x+2)\cdot(x-4) \qquad g(x) = (x+1)\cdot(x-2)\cdot(x+4)$$
$$h(x) = (x-1)\cdot(x-2)\cdot(x-4) \qquad k(x) = (x+1)\cdot(x-2)\cdot(x-4)$$



Sol

a) roots (x-intercepts): $x=1$, $x=2$, $x=4$

Polynomial $= (x-1)\cdot(x-2)\cdot(x-4) \Rightarrow h(x)$

b) roots: $x=-4$, $x=-2$, $x=1$

Polynomial $= (x+4)\cdot(x+2)\cdot(x-1) \Rightarrow$ none of above

c) roots: $x=-4$, $x=-1$, $x=2$

Polynomial $= (x+4)\cdot(x+1)\cdot(x-2) \Rightarrow g(x)$

d) roots: $x=-2$, $x=-1$, $x=4$

Polynomial $= (x+2)\cdot(x+1)\cdot(x-4) \Rightarrow$ none of above

e) roots: $x=-2$, $x=1$, $x=4$

Polynomial $= (x+2)\cdot(x-1)\cdot(x-4) \Rightarrow f(x)$

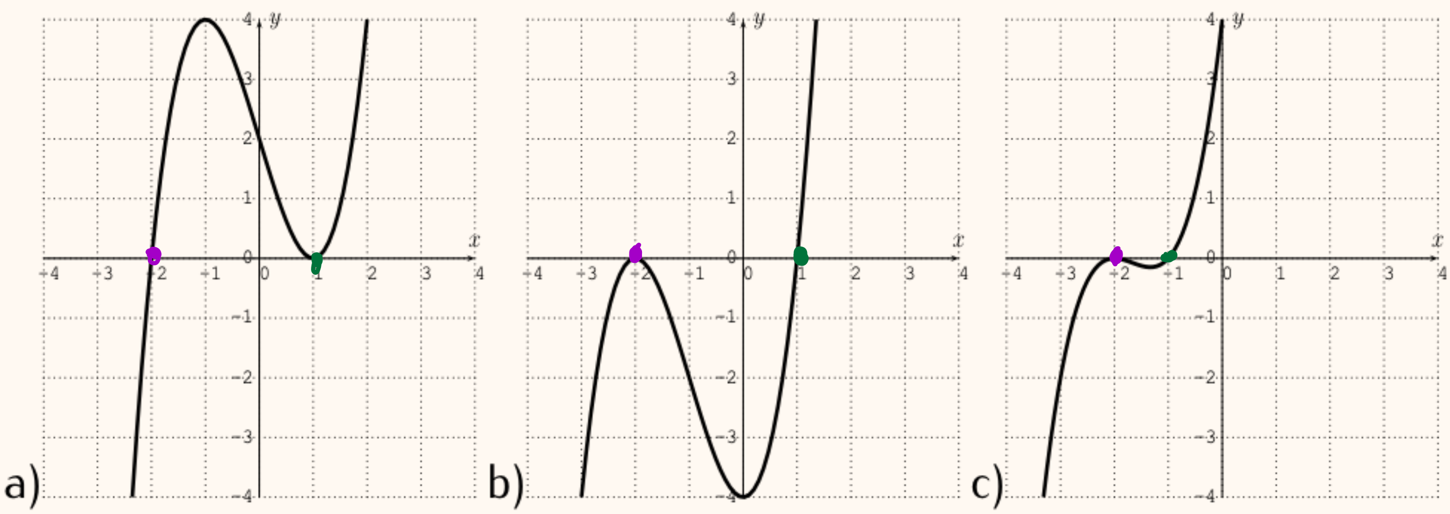
f) roots $x=-1$, $x=2$, $x=4$

Polynomial $= (x+1)\cdot(x-2)\cdot(x-4) \Rightarrow k(x)$.

Exercise 8.3

For each of the polynomials f, g, h, and k, find the corresponding graph from (a)-(f) below.

$$f(x) = (x+1)\cdot(x+2)^2 \qquad g(x) = -(x+1)\cdot(x-2)^2$$
$$h(x) = -(x-1)^2\cdot(x+2) \qquad k(x) = (x-1)\cdot(x+2)^2$$



Sol:

a) Roots: $x=-2$, $x=1$, $x=1$ (multiplicity $=2$)

Polynomial $= +(x+2)\cdot(x-1)^2 \Rightarrow$ None of above

(end behavior $(\swarrow,\nearrow) \Rightarrow$ leading coeff >0)

b) Roots: $x=-2$, $x=-2$ (multiplicity $=2$), $x=1$

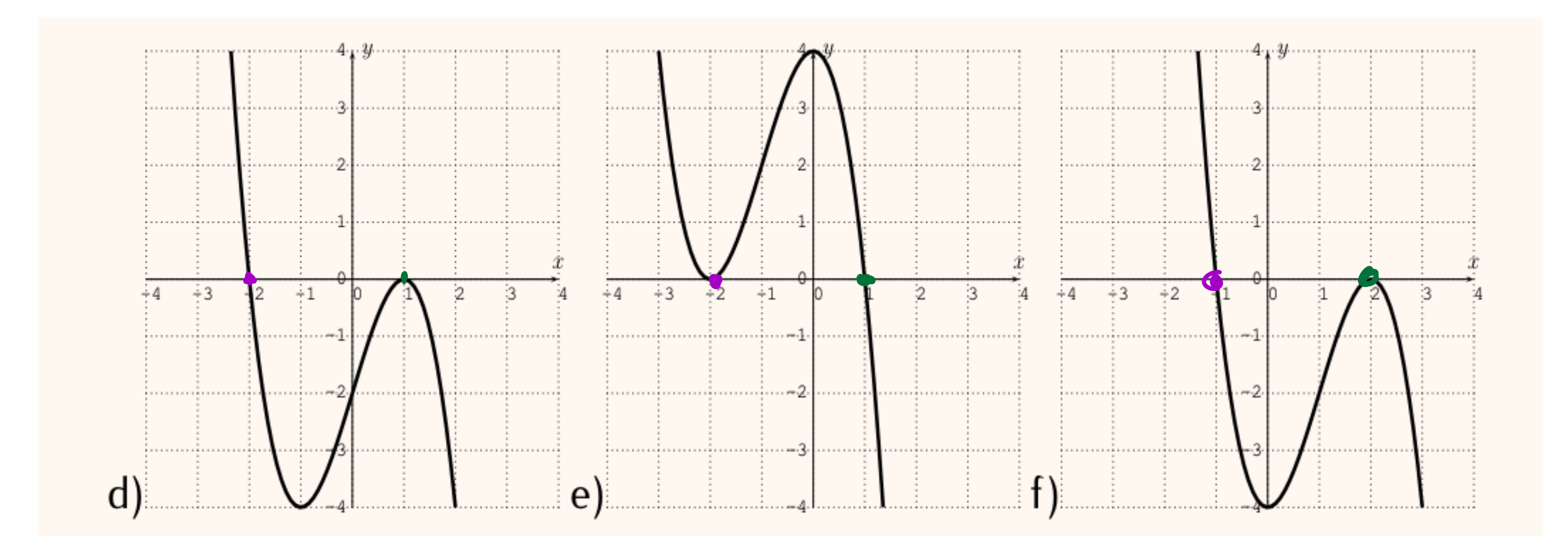
Polynomial $= +(x+2)^2\cdot(x-1) \Rightarrow k(x)$

(end behavior $(\swarrow,\nearrow) \Rightarrow$ leading coeff >0)

c) Roots: $x=-2$, $x=-2$ (multiplicity $=2$), $x=-1$

Polynomial $= +(x+2)^2\cdot(x+1) \Rightarrow f(x)$

(end behavior $(\swarrow, \nearrow) \Rightarrow$ leading coeff >0)



d) Roots: $x=-2$, $x=1$, $x=1$ (multiplicity $=2$)

Polynomial $= -(x+2)\cdot(x-1)^2 \Rightarrow h(x)$

(end behavior $(\uparrow, \downarrow) \Rightarrow$ leading coeff <0)

e) Roots $x=-2$, $x=-2$ (multiplicity $=2$), $x=1$

Polynomial $= -(x+2)^2(x-1) \Rightarrow$ None of above.

(end behavior $(\uparrow, \searrow) \Rightarrow$ Leading Coeff <0)

f) Roots $x=-1$, $x=2$, $x=2$

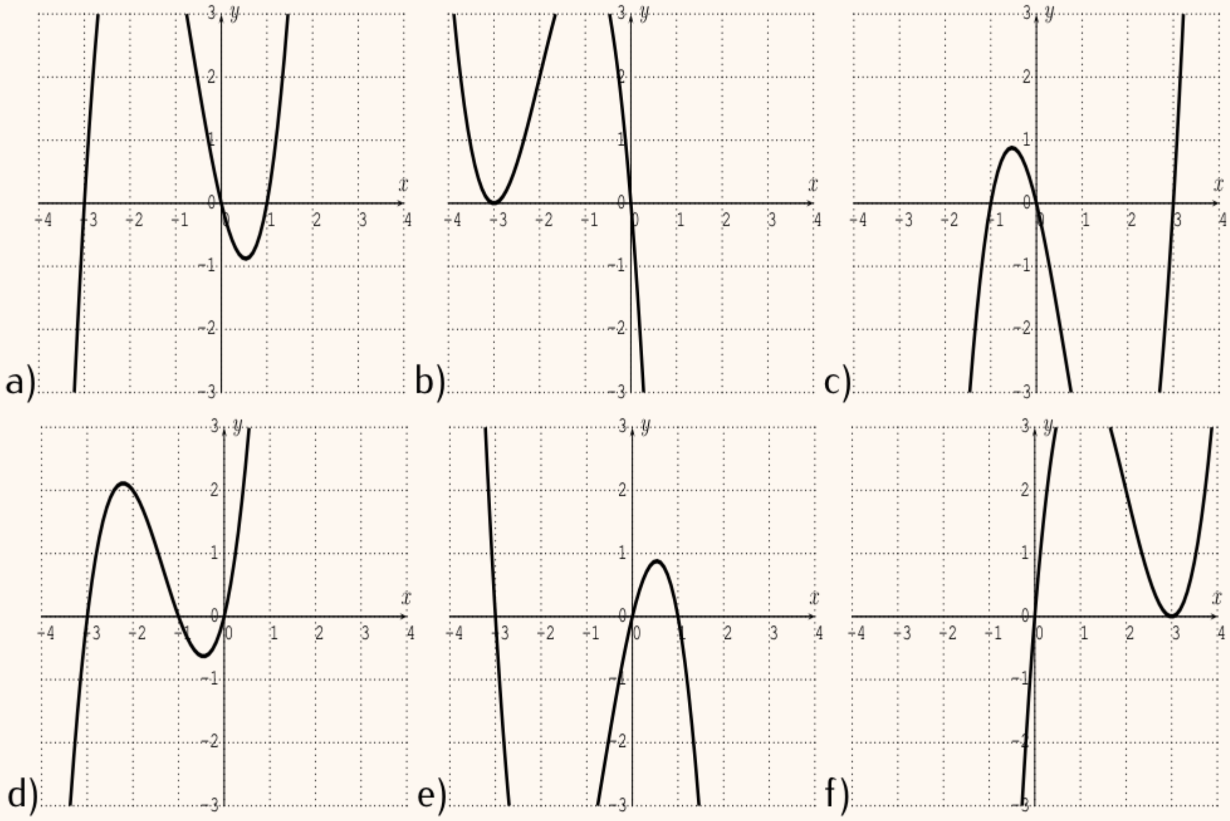
Polynomial $= -(x+1)\cdot(x-2)^2 \Rightarrow g(x)$

(end behavior $(\uparrow, \downarrow) \Rightarrow$ leading coeff <0)

Exercise 8.4

For each of the polynomials f, g, h, and k, find the corresponding graph from (a)–(f) below.

$$f(x) = x^3 + 4x^2 + 3x \qquad g(x) = -x^3 - 2x^2 + 3x$$
$$h(x) = x^{\cancel{2}\,3} - 2x^2 - 3x \qquad k(x) = -x^3 - 6x^2 - 9x$$



$\boxed{f(x)} = x^3+4x^2+3x = x(x^2+4x+3) = x(x+1)(x+3)$

the roots of $f(x) \Rightarrow f(x)=0 \Rightarrow x(x+1)(x+3)=0$

$\Rightarrow x=0,\ x=-1,\ x=-3$

End behavior x^3 < leading coeff $= 1 > 0$, degree $= 3$ (odd) $\Rightarrow (\swarrow, \nearrow)$

$\Rightarrow$ $\boxed{\text{graph (d)}}$

$\boxed{g(x)} = -x^3-2x^2+3x = -x(x^2+2x-3)$

$= -x(x+3)(x-1)$

The root(s) of $g(x)$: $g(x) = -x(x-1)(x+3) = 0$

$\Rightarrow \quad -x=0, \quad x-1=0, \quad x+3=0$

$\Rightarrow \quad x=0, \quad x=1, \quad x=-3$

End behavior $-x^3$ < leading coeff $= -1 < 0$, degree $= 3$ (odd) $\Rightarrow (\nwarrow, \searrow)$

$\Rightarrow$ $\boxed{\text{graph (e)}}$

$\boxed{h(x)} = x^3-2x^2-3x = x(x^2-2x-3) = x(x+1)(x-3)$

The roots of $h(x)$: $h(x) = x(x+1)(x-3) = 0$

$\Rightarrow \quad x=0$ or $x+1=0$ or $x-3=0$

$\Rightarrow \quad x=0$ or $x=-1$ or $x=3$

End behavior $1x^3$ < leading coeff $= 1 > 0$, degree $= 3$ (odd) $\Rightarrow (\swarrow, \nearrow)$

$\Rightarrow$ $\boxed{\text{graph (c)}}$

$\boxed{k(x)} = -x^3-6x^2-9x = -x(x^2+6x+9) = -x(x+3)^2$

The roots of $k(x)$: $k(x) = -x(x+3)^2 = 0$

$\Rightarrow \quad x=0$ or $x=-3$, $x=-3$ (multiplicity=2)

$\Rightarrow$ $\boxed{\text{graph (d)}}$

Exercise 8.5

Sketch a complete the graph of the function. Label all intercepts of the graph.

✓ a) $f(x) = x^3 + 4x^2 + x - 6$
✓ b) $f(x) = 2x^3 - 15x^2 + 34x - 24$
✓ c) $f(x) = x^3 - 16x - 21$

Sol

a) End behavior: leading term x^3 < leading Coeff $= 1 > 0$, degree $= 3$ (odd)

$\Rightarrow (\swarrow, \nearrow)$

Roots (x-intercepts): factors of -6 (constant term) $= \begin{matrix} 1, 2, 3, 6 \\ -1, -2, -3, -6 \end{matrix}$

education guess $x=1 \Rightarrow f(1) = 1^3 + 4 \cdot 1^2 + 1 - 6 = 0$

$\Rightarrow x = 1$ is a root

$\Rightarrow (x-1)$ is a factor of $f(x)$.

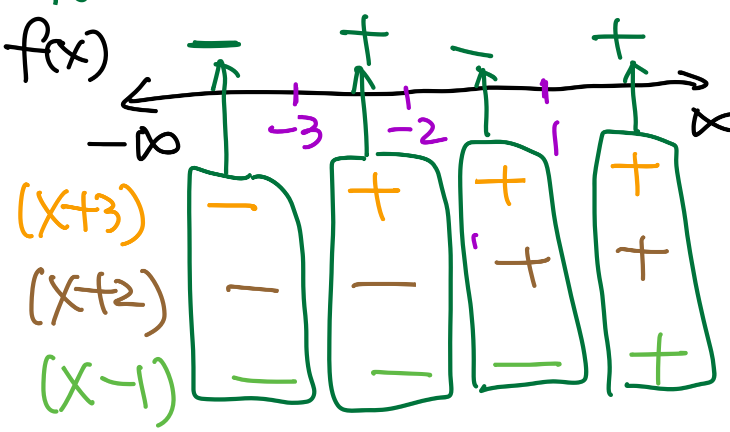
$$
\begin{array}{r|l}
 & x^2+5x+6 \\
\hline
x-1 & x^3+4x^2+x-6 \\
 & -(x^3-x^2) \\
\hline
 & 5x^2+x \\
 & -(5x^2-5x) \\
\hline
 & 6x-6 \\
 & -(6x-6) \\
\hline
 & 0
\end{array}
$$

$f(x) = (x-1) \cdot (x^2+5x+6)$

$= (x-1)(x+2)(x+3)$

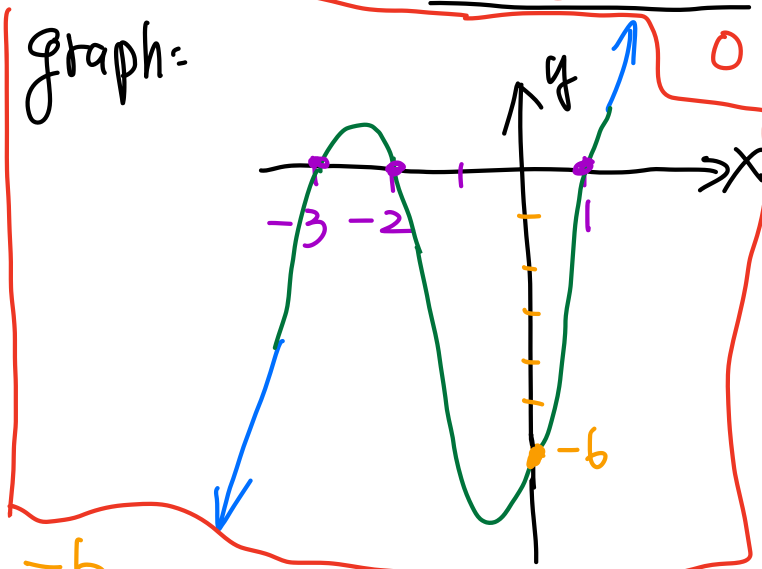
$f(x)$ has 3 roots: $x=1$, $x=-2$, $x=-3$

Number line:



y-intercept: ($x=0$, find $f(0)$)

$f(0) = 0^3 + 4 \cdot 0^2 + 0 - 6 = -6$



b) $f(x) = 2x^3 - 15x^2 + 34x - 24$

i) End behavior: leading term $= 2x^3$ $\nearrow$ leading coeff $= 2 > 0$; $\searrow$ degree $= 3$ (odd)

$\Rightarrow (\swarrow, \nearrow)$

ii) Roots: factors of constant term "-24": $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 8, \pm 12, \pm 24 \cdots$

education guess:

① $x = 1 \Rightarrow f(1) = 2\cdot 1 - 15\cdot 1 + 34\cdot 1 - 24 = -3 \neq 0 \Rightarrow$ not a root

② $x = -1 \Rightarrow f(-1) = 2\cdot(-1)^3 - 15\cdot(-1)^2 + 34\cdot(-1) - 24$

$= -2 - 15 - 34 - 24 \neq 0 \Rightarrow$ Not a root.

③ $x = 2, \Rightarrow f(2) = 2\cdot 2^3 - 15\cdot 2^2 + 34\cdot 2 - 24$

$= 16 - 60 + 68 - 24 = 0 \Rightarrow x = 2$ is a root!

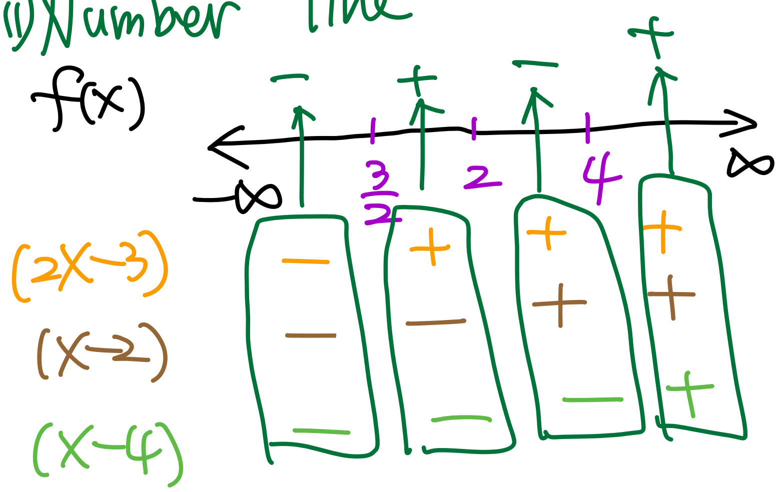
$$
\begin{array}{r|l}
 & 2x^2 - 11x + 12 \\
\hline
x-2 & 2x^3 - 15x^2 + 34x - 24 \\
 & -(2x^3 - 4x^2) \\
\hline
 & 11x^2 + 34x \\
 & -(11x^2 + 22x) \\
\hline
 & 12x - 24 \\
 & -(12x - 24) \\
\hline
 & 0
\end{array}
$$

$f(x) = (x-2)(2x^2 - 11x + 12)$

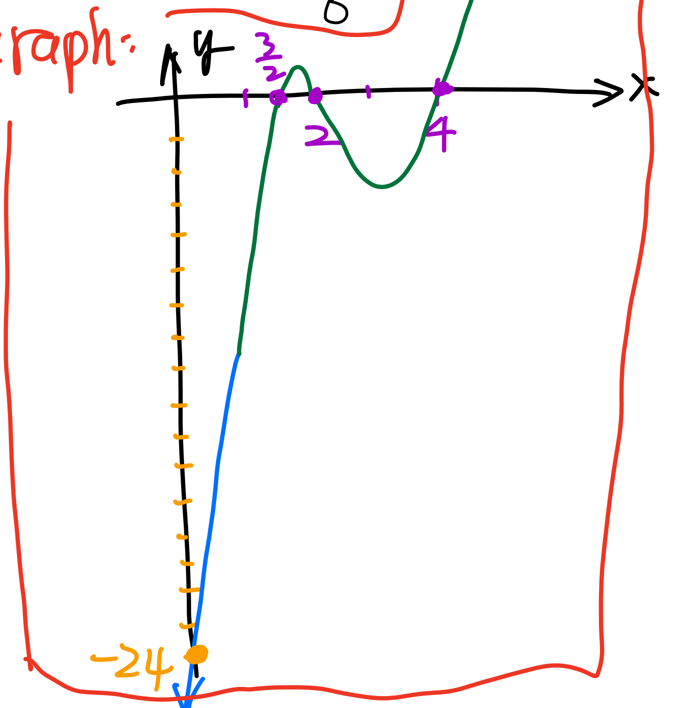
$= (x-2)(x-4)(2x-3)$

$f(x)$ has 3 roots: $x = \frac{3}{2}, 2, 4$

iii) Number line



graph:



iv) y-intercept:

$f(0) = 2\cdot 0^3 - 15\cdot 0^2 + 34\cdot 0 - 24 = -24$

c) $f(x)=x^3-16x-21$

i) End behavior: leading term $1x^3$ — leading coeff $=1>0$, degree $=3$ (odd) $\Rightarrow (\swarrow, \nearrow)$

ii) roots: factors of constant term "-21": $\pm1, \pm3, \pm7, \pm21$

education guess:

① $x=1$, $f(1)=1^3-16\cdot1-21\neq0$

② $x=-1$, $f(-1)=(-1)^3-16(-1)-21=-1+16-21\neq0$

③ $x=3$, $f(3)=3^3-16\cdot3-21\neq0$

④ $x=-3$, $f(-3)=(-3)^3-16\cdot(-3)-21=-27+48-21=0$

$\Rightarrow x=-3$ is a root.

$f(x)=(x+3)(x^2-3x-7)=0$

$$
\begin{array}{r l}
 & x^2-3x-7 \\
x+3 & \overline{)\, x^3+0x^2-16x-21} \\
 & -(x^3+3x^2) \\
 & \quad -3x^2-16x \\
 & \quad -(-3x^2-9x) \\
 & \qquad -7x-21 \\
 & \qquad -(-7x-21) \\
 & \qquad\quad 0
\end{array}
$$

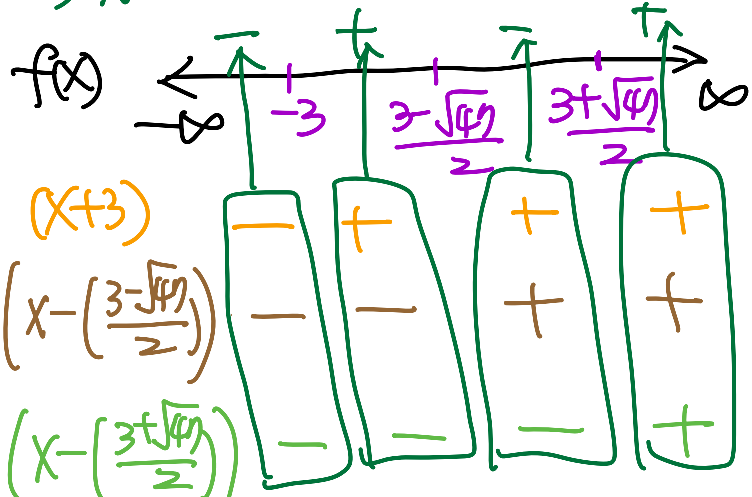
$\Rightarrow x+3=0$ or $x^2-3x-7=0$

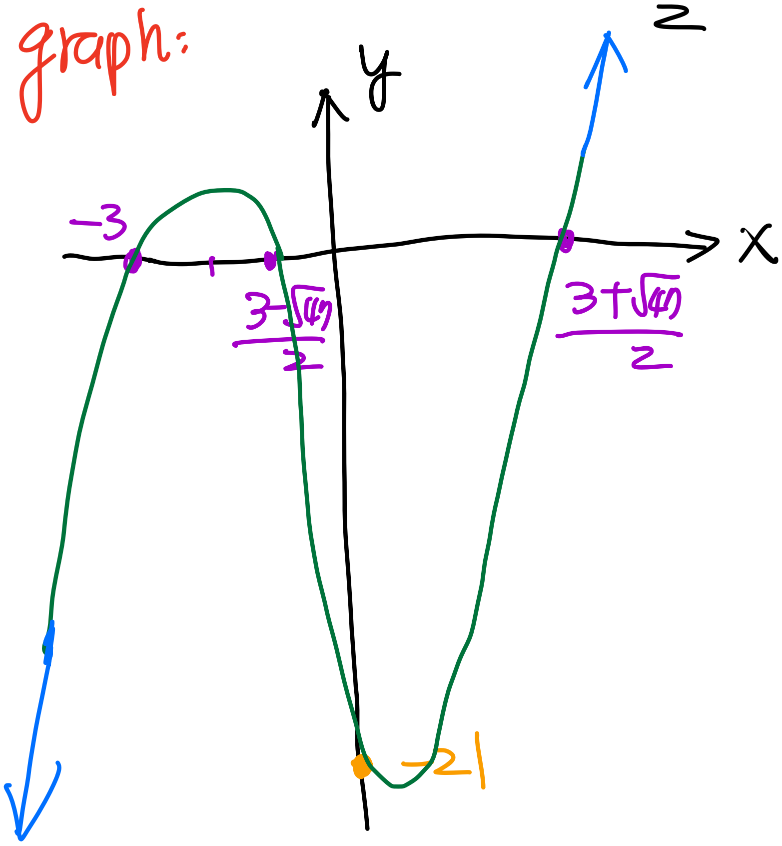
$\Rightarrow x=-3$ or $x=\frac{3\pm\sqrt{9+28}}{2}$

$=\frac{3\pm\sqrt{47}}{2}$

graph:

iii) Number line





iv) y-intercept

$f(0)=0^3-16\cdot0-21=-21$

Exercise 8.7

Find all roots and factor the polynomial completely.

a) $f(x) = x^3 - 5x^2 + 2x + 8$

b) $f(x) = x^3 + 7x^2 + 7x - 15$

c) $f(x) = x^3 + 9x^2 + 26x + 24$

d) $f(x) = x^3 + 4x^2 - 11x + 6$

e) $f(x) = 3x^3 + 13x^2 - 52x + 28$

f) $f(x) = 6x^3 - 5x^2 - 13x - 2$

g) $f(x) = 6x^3 - x^2 - 31x - 10$

h) $f(x) = x^3 - 7x^2 + 13x - 3$

i) $f(x) = x^3 + 2x^2 - 11x + 8$

j) $f(x) = 2x^3 + 7x^2 + 5x - 2$

k) $f(x) = 3x^3 - 10x^2 - 4x + 21$

b) $f(x) = x^3 + 7x^2 + 7x - 15$,

education guess: the factor of constant term "-15"

$\pm 1, \pm 3, \pm 5, \pm 15$

Check which one is a root of f(x).

① $x=1$, $f(1) = 1+7+7-15 = 0$ $\Rightarrow$ $x=1$ is a root!

$f(x) = (x-1)\cdot(x^2+8x+15) = (x-1)(x+3)(x+5) = 0$

$$
\begin{array}{r l}
 & x^2 + 8x + 15 \\
x-1 \,\big) & x^3 + 7x^2 + 7x - 15 \\
 & -(x^3 - x^2) \\ \hline
 & 8x^2 + 7x \\
 & -(8x^2 - 8x) \\ \hline
 & 15x - 15 \\
 & -(15x - 15) \\ \hline
 & 0
\end{array}
$$

$\Rightarrow$ $\boxed{x=1 \text{ or } x=-3 \text{ or } x=-5}$

g) $f(x)=6x^3-x^2-31x-10$

education guess: the factor of the constant term "-10"

$\pm 1, \pm 2, \pm 5, \pm 10,$

$(\pm\frac{1}{2}, \pm\frac{5}{2}, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{5}{3}, \pm\frac{10}{3},$

$\pm\frac{1}{6}, \pm\frac{5}{6},)$

Check roots

① $x=1,\ f(1)=6\cdot 1^3-1^2-31-10\neq 0$

② $x=-1,\ f(-1)=6(-1)^3-(-1)^2-31(-1)-10\neq 0$

③ $x=2,\ f(2)=6\cdot 2^3-2^2-31\cdot 2-10\neq 0$

④ $\boxed{x=-2},\ f(-2)=6\cdot(-2)^3-(-2)^2-31\cdot(-2)-10$

$=-48-4+62-10=0$ this is a root!

$f(x)=(x+2)(6x^2-13x-5)=(x+2)(2x-1)(3x-5)$

$=0$

$$\begin{array}{r l}
 & 6x^2-13x-5 \\
x+2 \,\big) & 6x^3-x^2-31x-10 \\
 & -(6x^3+12x^2) \\ \hline
 & -13x^2-31x \\
 & -(-13x^2-26x) \\ \hline
 & -5x-10 \\
 & -(-5x-10) \\ \hline
 & 0
\end{array}$$

$\boxed{x=-2 \text{ or } \frac{1}{2} \text{ or } \frac{5}{3}}$

i) $f(x)=x^3+2x^2-11x+8$

education guess: the factors of 8:

$\pm 1, \pm 2, \pm 4, \pm 8$

Check roots:

① $\boxed{x=1,}$ $f(1)=1^3+2\cdot 1^2-11\cdot 1+8=0$ this is a root!

$f(x)=(x-1)(x^2+3x-8)=0$

$\Rightarrow x-1=0$ or $x^2+3x-8=0$

$$\begin{array}{r|rrrr} 1 & 1 & 2 & -11 & 8 \\ & & 1 & 3 & -8 \\ \hline & 1 & 3 & -8 & \boxed{0} \end{array}$$

$\Rightarrow x^2+3x-8$

$\Rightarrow \boxed{x=1}$ or $x=\frac{-3\pm\sqrt{9+32}}{2}$

$\boxed{=\frac{-3\pm\sqrt{41}}{2}}$

j) $f(x)=2x^3+7x^2+5x-2$

education guess: the factor of "-2"

$\pm 1, \pm 2 \quad (\pm\frac{1}{2})$

check roots:

① $x=1$, $f(1)=2\cdot 1+7\cdot 1+5\cdot 1-2\neq 0$

② $x=-1$, $f(-1)=2\cdot(-1)^3+7\cdot(-1)^2+5(-1)-2$

$=-2+7-5-2\neq 0$

③ $x=2$, $f(2)=2\cdot 2^3+7\cdot 2^2+5\cdot 2-2\neq 0$

④ $\boxed{x=-2,}$ $f(-2)=2\cdot(-2)^3+7\cdot(-2)^2+5\cdot(-2)-2$

$=-16+28-10-2=0$

this is a root!

$f(x)=(x+2)(2x^2+3x-1)=0$

$$\begin{array}{r|rrrr} -2 & 2 & +7 & +5 & -2 \\ & & -4 & -6 & +2 \\ \hline & 2 & +3 & -1 & \boxed{0} \end{array}$$

$2x^2+3x-1$

$x+2=0$ or $2x^2+3x-1=0$

$\Rightarrow x=-2$ or $x=\frac{-3\pm\sqrt{9+8}}{4}$

$=\frac{-3\pm\sqrt{17}}{4}$

Exercise 8.8

Graph the following polynomials without using the calculator.

a) $f(x) = (x+4)^2(x-5)$

End behavior: $f(x) = (x+4)(x+4)(x-5)$

leading term $= x \cdot x \cdot x = x^3 \Rightarrow (\downarrow, \uparrow)$

Roots: $f(x) = (x+4)^2(x-5) = 0$

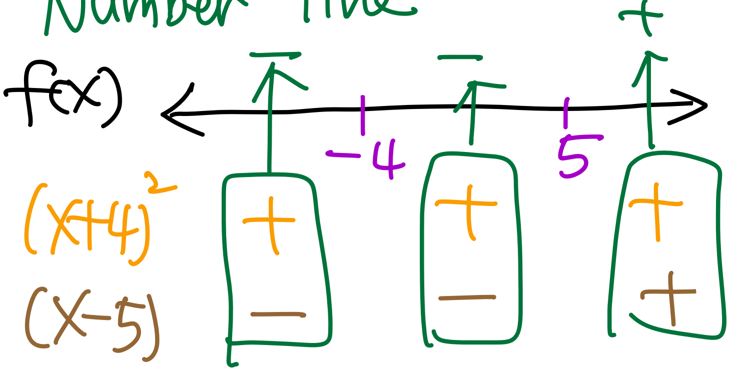
$x+4=0$ or $x+4=0$ or $x-5=0$

$\Rightarrow x=-4$ or $x=-4$ or $x=5$

y-intercept: $f(0) = (0+4)^2(0-5)$

$= 16 \cdot (-5) = -80$

Number line



graph: