Mat 1375 HW5 5.1 ~ 5.5

Exercise 5.1

Find $f+g$, $f-g$, $f\cdot g$ for the functions below. State their domain.

✓ a) $f(x) = x^2 + 6x$ and $g(x) = 3x - 5$
✓ b) $f(x) = x^3 + 5$ and $g(x) = 5x^2 + 7$
✓ c) $f(x) = 3x + 7\sqrt{x}$ and $g(x) = 2x^2 + 5\sqrt{x}$

Sol

a) $D_f = (-\infty, \infty)$, $D_g = (-\infty, \infty)$

domain

$$(f+g)(x) = f(x) + g(x) = (x^2+6x) + (3x-5) = x^2+6x+3x-5 = x^2+9x-5$$

$D_{f+g} = D_f \cap D_g = \{x \mid x \in (-\infty, \infty)\}$

$$(f-g)(x) = f(x) - g(x) = (x^2+6x) - (3x-5) = x^2+6x-3x+5 = x^2+3x+5$$

$D_{f-g} = D_f \cap D_g = \{x \mid x \in (-\infty, \infty)\}$

$$(f\cdot g)(x) = f(x)\cdot g(x) = (x^2+6x)\cdot(3x-5) = 3x^3+18x^2-5x^2-30x = 3x^3+13x^2-30x$$

$D_{f\cdot g} = D_f \cap D_g = \{x \mid x \in (-\infty, \infty)\}$

b) $D_f = (-\infty, \infty)$, $D_g = (-\infty, \infty)$

domain

$$(f+g)(x) = f(x) + g(x) = (x^3+5) + (5x^2+7) = x^3+5x^2+12$$

$D_{f+g} = D_f \cap D_g = \{x \mid x \in (-\infty, \infty)\}$

$$(f-g)(x) = f(x) - g(x) = (x^3+5) - (5x^2+7) = x^3-5x^2-2$$

$D_{f-g} = D_f \cap D_g = \{x \mid x \in (-\infty, \infty)\}$

$(f\cdot g)(x) = f(x)\cdot g(x) = (x^3+5)\cdot(5x^2+7)$

$= 5x^5+7x^3+25x^2+35$

$D_{f\cdot g} = D_f \cap D_g$

$= \{x \mid x\in(-\infty,\infty)\}$

c) $D_f = [0,\infty)$, $D_g = [0,\infty)$

(Since $f(x) = 3x+7\sqrt{x}$ with a square root, then any $x<0$ will not be in the domain of f, so $D_f = [0,\infty)$.

Similary, $g(x)$ has a term of $5\sqrt{x}$, so $D_g = [0,\infty)$ domain)

$(f+g)(x) = f(x)+g(x) = (3x+7\sqrt{x})+(2x^2+5\sqrt{x})$

$= 2x^2+3x+12\sqrt{x}$

$D_{f+g} = D_f \cap D_g$

$= \{x \mid x\in[0,\infty)\}$

$(f-g)(x) = f(x)-g(x) = (3x+7\sqrt{x})-(2x^2+5\sqrt{x})$

$= -2x^2+3x+2\sqrt{x}$

$D_{f-g} = D_f \cap D_g$

$= \{x \mid x\in[0,\infty)\}$

$(f\cdot g)(x) = f(x)\cdot g(x) = (3x+7\sqrt{x})\cdot(2x^2+5\sqrt{x})$

$= 6x^3+14x^2\sqrt{x}+15x\sqrt{x}+35(\sqrt{x})^2$

$= 6x^3+14x^2\sqrt{x}+15x\sqrt{x}+35x$

$D_{f\cdot g} = D_f \cap D_g$

$= \{x \mid x\in[0,\infty)\}$

Exercise 5.2

Find $\frac{f}{g}$, and $\frac{g}{f}$ for the functions below. State their domain.

a) $f(x) = 3x + 6$ and $g(x) = 2x - 8$

b) $f(x) = x + 2$ and $g(x) = x^2 - 5x + 4$

Sol

a) $D_f = (-\infty, \infty)$, $D_g = (-\infty, \infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{3x+6}{2x-8}$$

Domain

$D_{\frac{f}{g}} := D_f \cap D_g$ but $g(x) \neq 0$

$\Rightarrow (-\infty, \infty)$ but $g(x) \neq 0$

$\Rightarrow (-\infty, \infty)$ but $2x - 8 \neq 0$

$\Rightarrow (-\infty, \infty)$ but $x \neq 4$

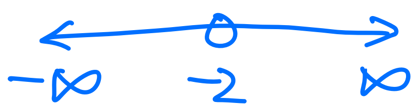
$D_{\frac{f}{g}} = \{x \mid x \in (-\infty, 4) \cup (4, \infty)\}$

$$\left(\frac{g}{f}\right)(x) = \frac{g(x)}{f(x)} = \frac{2x-8}{3x+6}$$

$D_{\frac{g}{f}} = D_f \cap D_g$ but $f(x) \neq 0$

$\Rightarrow (-\infty, \infty)$ but $3x + 6 \neq 0$

$\Rightarrow (-\infty, \infty)$ but $x \neq -2$



$D_{\frac{g}{f}} = \{x \mid x \in (-\infty, 2) \cup (2, \infty)\}$

b) $D_f = (-\infty, \infty)$, $D_g = (-\infty, \infty)$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x+2}{x^2 - 5x + 4}$$

Domain

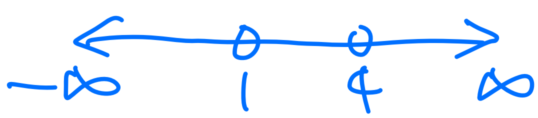
$D_{\frac{f}{g}} := D_f \cap D_g$ but $g(x) \neq 0$

$\Rightarrow (-\infty, \infty)$, but $x^2 - 5x + 4 \neq 0$

$\Rightarrow (-\infty, \infty)$, but $(x-1)(x-4) \neq 0$

$\Rightarrow (-\infty, \infty)$, but $x - 1 \neq 0$ and $x - 4 \neq 0$

$\Rightarrow (-\infty, \infty)$, but $x \neq 1$ and $x \neq 4$



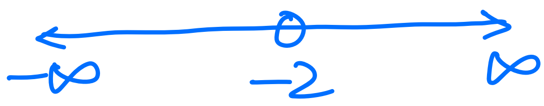
$D_{\frac{f}{g}} = \{x \mid x \in (-\infty, 1) \cup (1, 4) \cup (4, \infty)\}$

$$\left(\frac{g}{f}\right)(x)=\frac{g(x)}{f(x)}=\frac{x^2-5x+4}{x+2}$$

$D_{\frac{g}{f}} := D_g \cap D_f$ but $f(x) \neq 0$

$\Rightarrow (-\infty,\infty)$ but $x+2 \neq 0$

$\Rightarrow (-\infty,\infty)$ but $x \neq -2$



, $D_{\frac{g}{f}} = \{x \mid x \in (-\infty,-2) \cup (-2,\infty)\}$

Exercise 5.3

Let $f(x) = 2x - 3$ and $g(x) = 3x^2 + 4x$. Find the following compositions:

a) $f(g(2))$ b) $g(f(2))$ c) $f(f(5))$
d) $f(5g(-3))$ e) $g(f(2)-2)$ f) $f(f(3)+g(3))$

Sol.

a) $g(2) = 3\cdot(2)^2 + 4\cdot(2) = 3\cdot 4 + 8 = 20$

$f(g(2)) = f(20) = 2\cdot(20) - 3 = 40 - 3 = \boxed{37}$

b) $f(2) = 2\cdot(2) - 3 = 4 - 3 = 1$

$g(f(2)) = g(1) = 3\cdot(1)^2 + 4\cdot(1) = \boxed{7}$

c) $f(5) = 2(5) - 3 = 10 - 3 = 7$

$f(f(5)) = f(7) = 2\cdot(7) - 3 = 14 - 3 = \boxed{11}$

d) $g(-3) = 3(-3)^2 + 4\cdot(-3) = 3\cdot 9 - 12 = 15$

$5\cdot g(-3) = 5\cdot 15 = 75$

$f(5g(-3)) = f(75) = 2(75) - 3 = 150 - 3 = \boxed{147}$

Exercise 5.4

Find the composition $(f \circ g)(x)$ for the following functions:

a) $f(x) = 3x - 5$ and $g(x) = 2x + 3$

b) $f(x) = x^2 + 2$ and $g(x) = x + 3$

c) $f(x) = x^2 - 3x + 2$ and $g(x) = 2x + 1$

Sol:

replace "x" from f(x) by "g(x)"

a) $(f \circ g)(x) = f(g(x)) = f(2x+3)$

$= 3(2x+3) - 5$

$= 6x + 9 - 5 = \boxed{6x+4}$

b) $(f \circ g)(x) = f(g(x)) = f(x+3)$

$= (x+3)^2 + 2$

$= x^2 + 6x + 9 + 2 = \boxed{x^2+6x+11}$

* $(x+3)(x+3)$

$= x^2 + 3x + 3x + 9$

$= x^2 + 6x + 9$

c) $(f \circ g)(x) = f(g(x)) = f(2x+1)$

$= (2x+1)^2 - 3(2x+1) + 2$

$= 4x^2 + 4x + 1 - 6x - 3 + 2$

$= \boxed{4x^2 - 2x}$

* $(2x+1)^2$

$= (2x+1)(2x+1)$

$= 4x^2 + 2x + 2x + 1$

$= 4x^2 + 4x + 1$

Exercise 5.5

Find the compositions

$$(f \circ g)(x), \quad (g \circ f)(x), \quad (f \circ f)(x), \quad (g \circ g)(x)$$

for the following functions:

a) $f(x) = 2x + 4$ and $g(x) = x - 5$

b) $f(x) = x + 3$ and $g(x) = x^2 - 2x$

Sol

a) $(f\circ g)(x) = f(g(x)) = f(x-5)$ replace "x" in f(x) by "g(x)"

$= 2(x-5)+4 = 2x-10+4 = \boxed{2x-6}$

$(g\circ f)(x) = g(f(x)) = g(2x+4)$ replace "x" in g(x) by "f(x)"

$= (2x+4)-5 = \boxed{2x-1}$

$(f\circ f)(x) = f(f(x)) = f(2x+4)$

$= 2(2x+4)+4 = 4x+8+4 = \boxed{4x+12}$

$(g\circ g)(x) = g(g(x)) = g(x-5)$

$= (x-5)-5 = \boxed{x-10}$

b) $(f\circ g)(x) = f(g(x)) = f(x^2-2x)$

$= (x^2-2x)+3 = \boxed{x^2-2x+3}$

$(g\circ f)(x) = g(f(x)) = g(x+3)$

$= (x+3)^2-2(x+3)$

$$= x^2+6x+9-2x-6 = \boxed{x^2+4x+3}$$

$$(f\circ f)(x) = f(f(x)) = f(x+3)$$

$$= (x+3)+3 = \boxed{x+6}$$

$$(g\circ g)(x) = g(g(x)) = g(x^2-2x)$$

$$= (x^2-2x)^2 - 2(x^2-2x)$$

$$= (x^2-2x)(x^2-2x) - 2x^2+4x$$

$$= x^4-2x^3-2x^3+4x^2-2x^2+4x$$

$$= \boxed{x^4-4x^3+2x^2+4x}$$