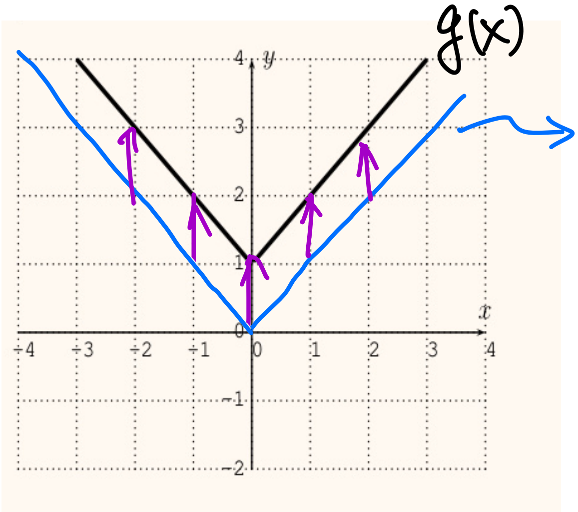
Mat 1375 HW4

Exercise 4.8

Find a possible formula of the graph displayed below.

Sol

a)

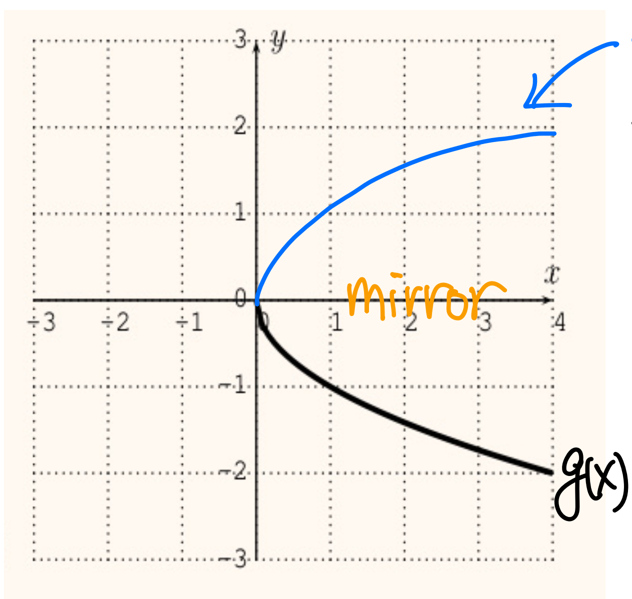


$f(x)=|x|$

The given graph $g(x)$ is the absolute function $f(x)=|x|$ shifted up by 1 unit $\Rightarrow$ $\boxed{g(x)=|x|+1}$

b)

$f(x)=\sqrt{x}$

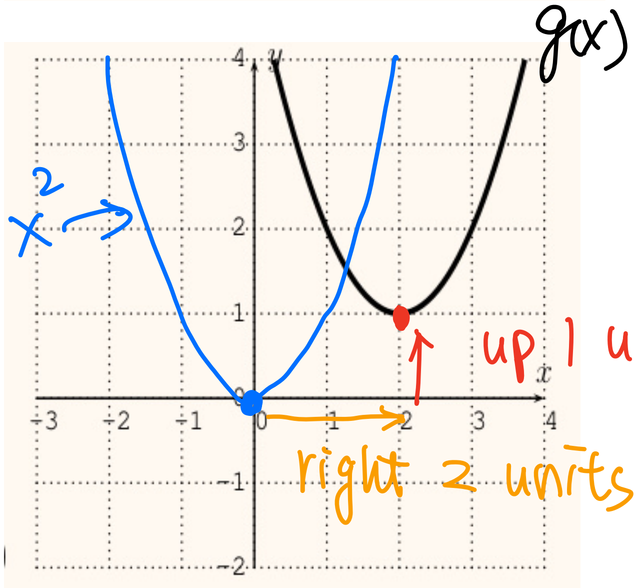


The given graph $g(x)$ is the reflection of $f(x)=\sqrt{x}$ about x-axis

$\Rightarrow$ $\boxed{g(x)=-\sqrt{x}}$

c)

$f(x)=x^2$



$f(x)=x^2$

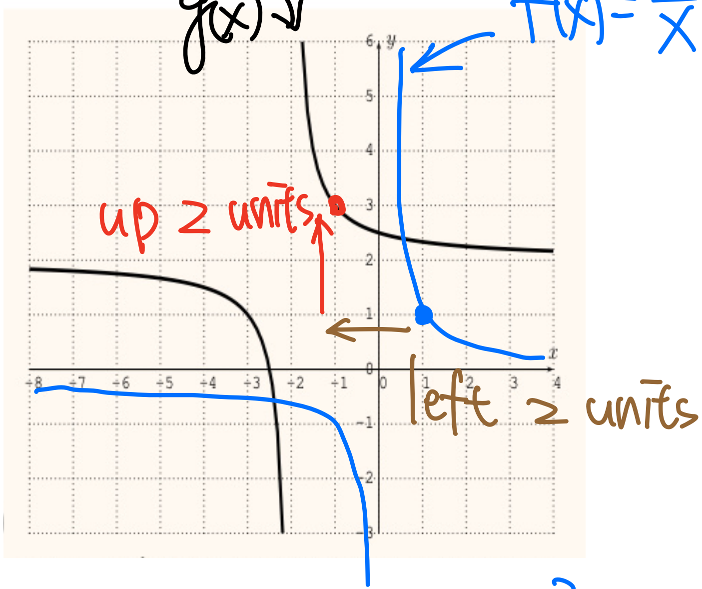
$\Downarrow$ shift 2 units to the right

$y=(x-2)^2$

$\Downarrow$ up 1 unit

$\boxed{g(x)=(x-2)^2+1}$

d)



$f(x) = \frac{1}{x}$

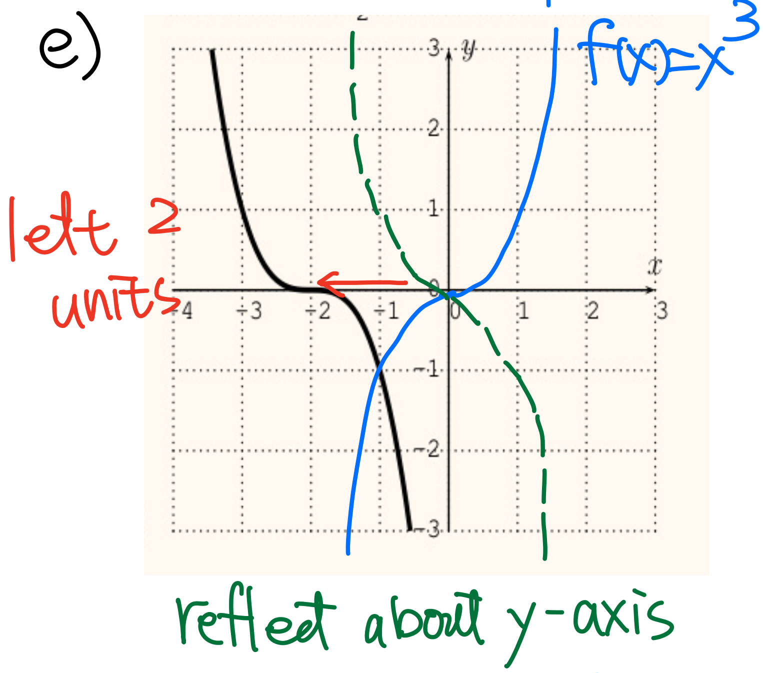
$\Downarrow$ shift 2 units to the left

$y = \frac{1}{x+2}$

$\Downarrow$ up 2 units

$$\boxed{g(x) = \frac{1}{x+2} + 2}$$

e)



$f(x) = x^3$

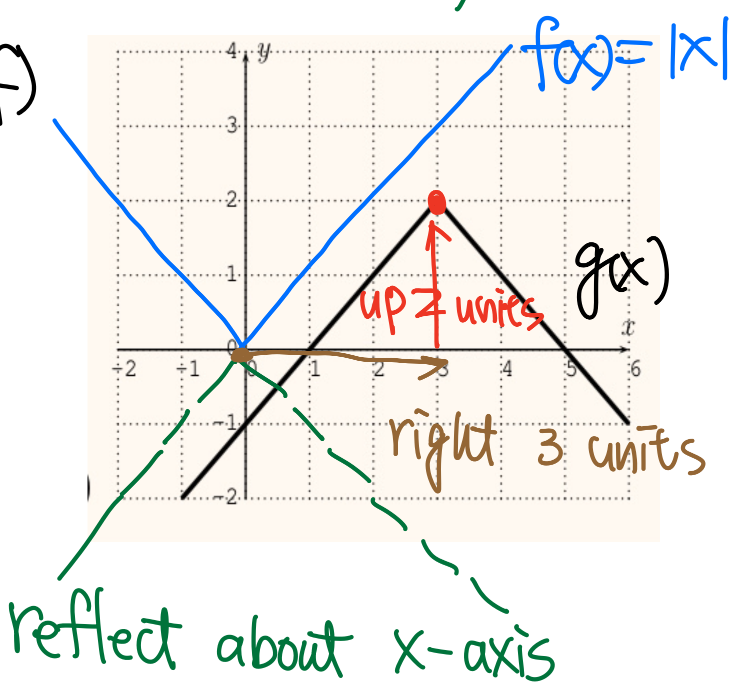
$\Downarrow$ reflect about y-axis

$y = (-x)^3 = -x^3$

$\Downarrow$ shift 2 unit to the left

$$\boxed{g(x) = -(x+2)^3}$$

f)



$f(x) = |x|$

$\Downarrow$ reflect about x-axis

$y = -|x|$

$\Downarrow$ shift 3 units to the right

$y = -|x-3|$

$\Downarrow$ up 2 units

$$\boxed{g(x) = -|x-3| + 2}$$

Exercise 4.9

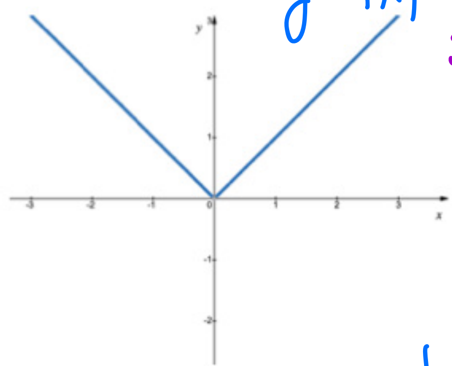
Sketch the graph of the function based on the basic graphs and their transformation described in Section 4.3. Confirm your answer by graphing the function with the graphing calculator.

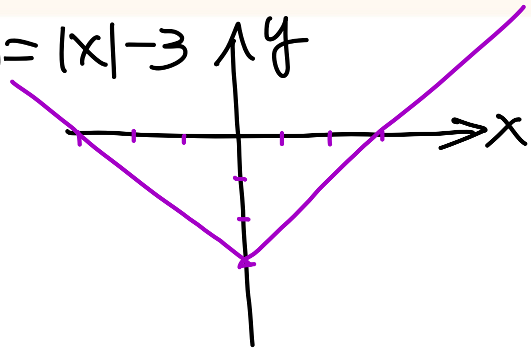
a) $f(x) = |x| - 3$ b) $f(x) = \frac{1}{x+2}$
c) $f(x) = -x^2$ d) $f(x) = (x-1)^3$
e) $f(x) = \sqrt{-x}$ f) $f(x) = 4 \cdot |x-3|$
g) $f(x) = -\sqrt{x} + 1$ h) $f(x) = (\frac{1}{2} \cdot x)^2 + 3$

Sol

a)

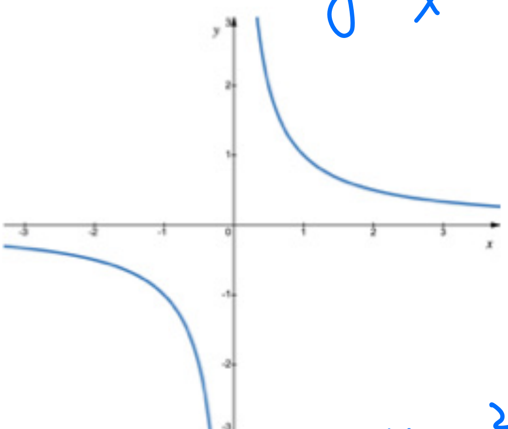
$y = |x| \xrightarrow{\text{shift down 3 units}} f(x) = |x| - 3$

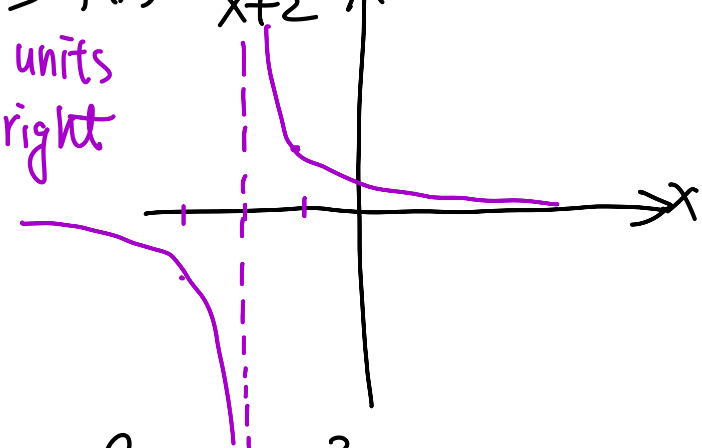




b)

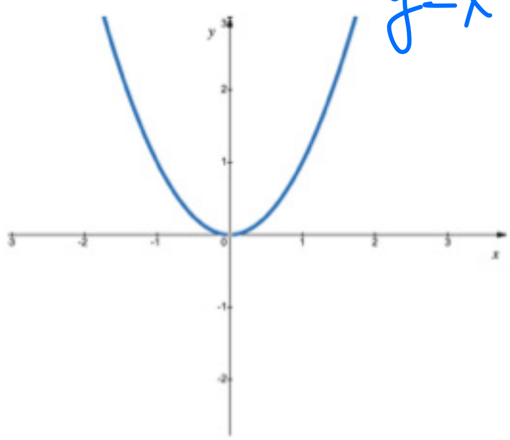
$y = \frac{1}{x} \xrightarrow{\text{shift 2 units to the right}} f(x) = \frac{1}{x+2}$

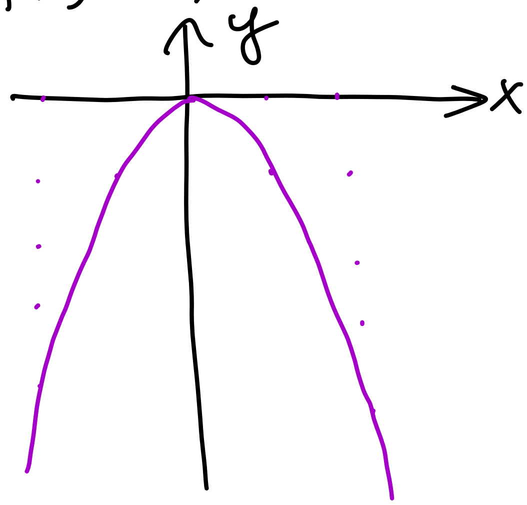




c)

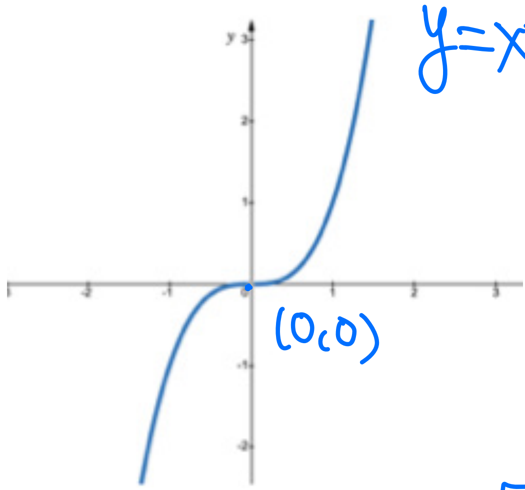
$y = x^2 \xrightarrow{\text{reflect about x-axis}} f(x) = -x^2$

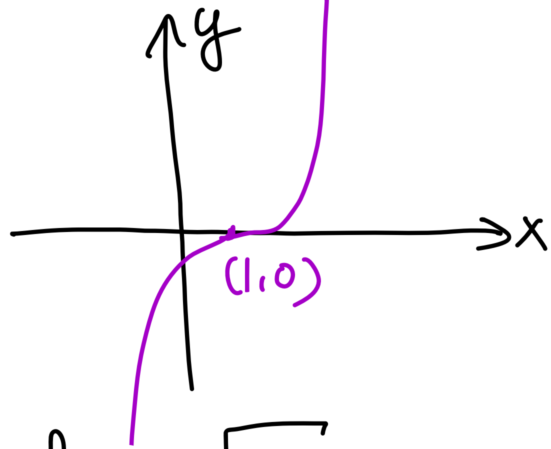




d)

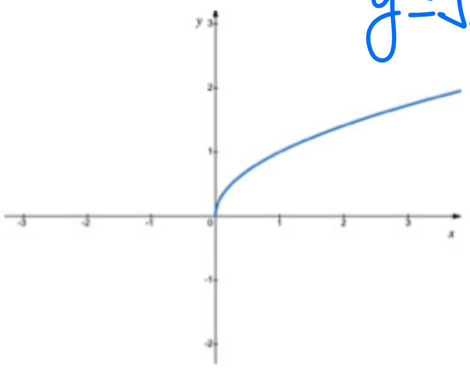
$y=x^3$ shift 1 unit to the right $f(x)=(x-1)^3$

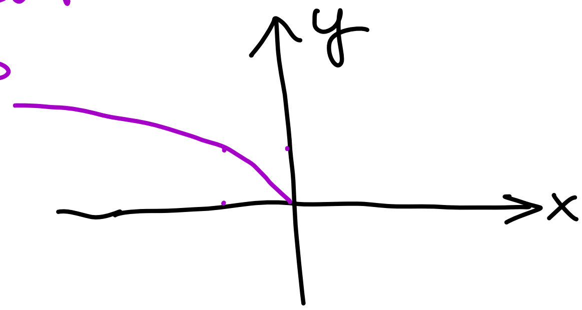




e)

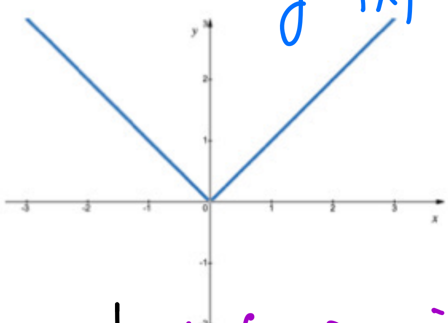
$y=\sqrt{x}$ reflect about y-axis $f(x)=\sqrt{-x}$





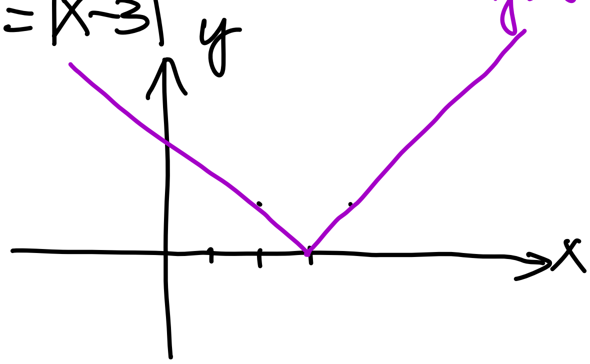
f)

$y=|x|$



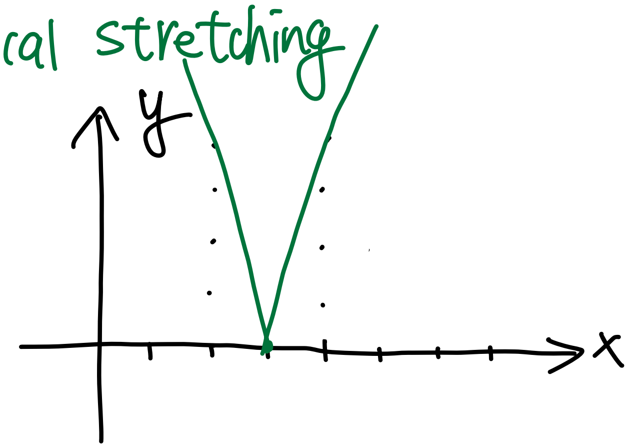
shift 3 units to the right

$y=|x-3|$



Vertical stretching

$f(x)=4\cdot|x-3|$



Exercise 4.10

Consider the graph of $f(x) = x^2 - 7x + 1$. Find the formula of the function that is given by performing the following transformations on the graph.

✓a) Shift the graph of f down by 4.
✓b) Shift the graph of f to the left by 3 units.
✓c) Reflect the graph of f about the x-axis.
✓d) Reflect the graph of f about the y-axis.
e) Stretch the graph of f away from the y-axis by a factor 3.
f) Compress the graph of f toward the y-axis by a factor 2.

Sol

a) Shift f down by 4 $\Rightarrow$ subtract 4 from $f(x)$

$$\Rightarrow y = x^2 - 7x + 1 - 4 \Rightarrow \boxed{y = x^2 - 7x - 3}$$

b) Shift f 3 units to the left $\Rightarrow$ replace x by $x+3$

$$\Rightarrow \boxed{y = (x+3)^2 - 7(x+3) + 1}$$

c) Reflect f about x-axis $\Rightarrow$ $f(x)$ is multiplied by -1.

$$\Rightarrow \boxed{y = -(x^2 - 7x + 1) = -x^2 + 7x - 1}$$

d) Reflect f about y-axis $\Rightarrow$ replace x by $-x$

$$\Rightarrow y = (-x)^2 - 7(-x) + 1$$

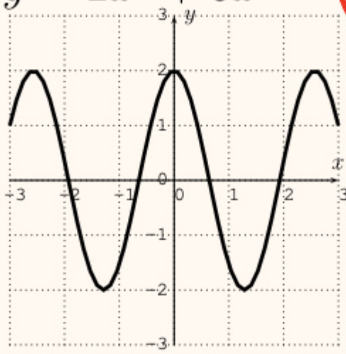
$$\Rightarrow \boxed{y = x^2 + 7x + 1}$$

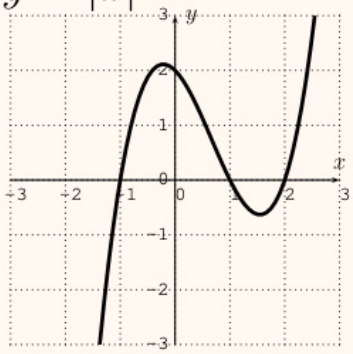
Exercise 4.12

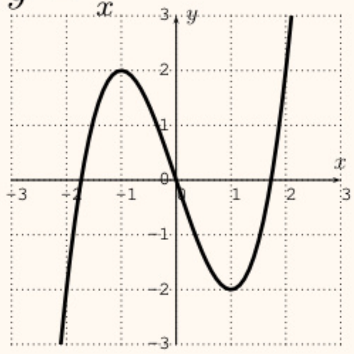
Determine if the function is even, odd, or neither.

a) $y = 2x^3$ b) $y = 5x^2$ c) $y = 3x^4 - 4x^2 + 5$

d) $y = 2x^3 + 5x^2$ e) $y = |x|$ f) $y = \frac{1}{x}$

g) 

h) 

i) 

Sol.

a) Let $f(x) = 2x^3$. So $-f(x) = -2x^3$

Replacing x by "$-x$", we have $f(-x) = 2(-x)^3 = 2\cdot(-x^3)$
$= -2x^3$

Since $f(-x) = -f(x)$, f is an odd function

b) Let $f(x) = 5x^2$.

Replacing x by "$-x$", we have $f(-x) = 5(-x)^2 = 5x^2$

Since $f(-x) = f(x)$, f is an even function.

c) Let $f(x) = 3x^4 - 4x^2 + 5$.

Replacing x by "$-x$", we have

$$f(-x) = 3(-x)^4 - 4(-x)^2 + 5 = 3x^4 - 4x^2 + 5$$

Since $f(-x) = f(x)$, f is an even function.

d) Let $f(x) = 2x^3 + 5x^2$.

Replacing x by "$-x$", we have

$$f(-x) = 2(-x)^3 + 5(-x)^2 = -2x^3 + 5x^2$$

Since $f(-x) \neq -f(x)$, $f(-x) \neq f(x)$, then $f(x)$ is neither odd nor even function.

e) Let $f(x) = |x|$

Replacing x by "$-x$", we have

$$f(-x) = |-x| = x$$

Since $f(-x) = f(x)$, then $f(x)$ is an even function.