Mat 1375 HW3.

Exercise 3.1

Find the slope and y-intercept of the line with the given data. Using the slope and y-intercept, write the equation of the line in slope-intercept form.

Sol

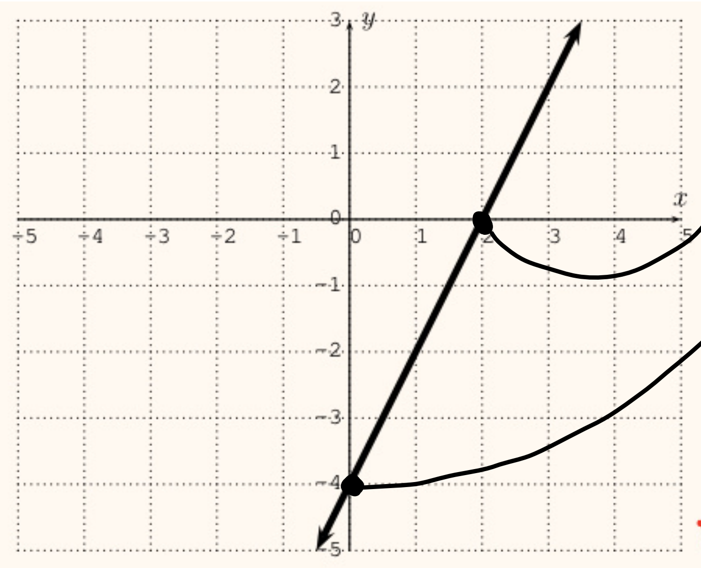
① Find two points from the graph:

$(2,0)$ and $(0,-4)$

② Let $(x_1, y_1) = (2, 0)$, $(x_2, y_2) = (0, -4)$

and find the slope of line by slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

a)



$$\text{slope} = \frac{-4-0}{0-2} = \frac{-4}{-2} = \frac{4}{2} = 2$$

③ Find the y-intercept point (the intersection point of the graph and y-axis)

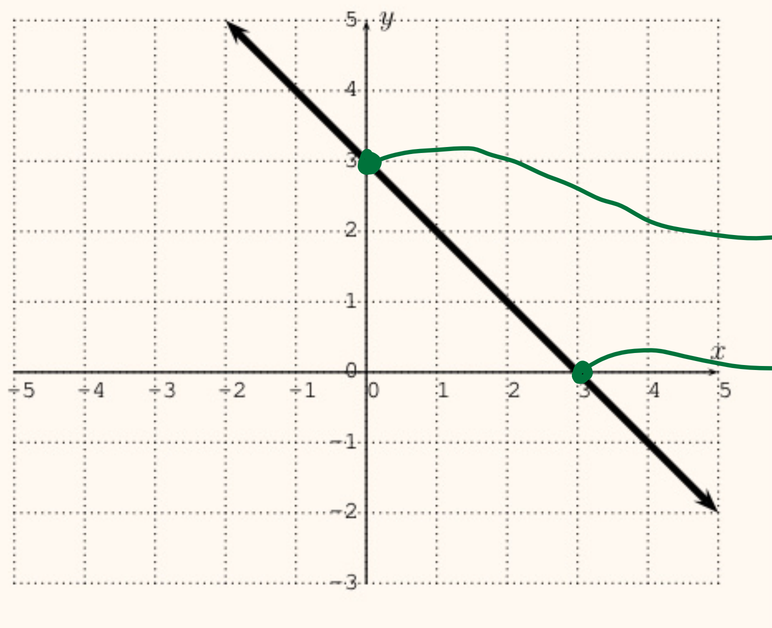
y-intercept is $(0, -4)$

④ Slope-intercept form: $y = (\text{slope}) \cdot x + b$

$y = 2x + b$ and y interept is $(0, -4)$

plug $(0, 4)$ into $y = 2x + b$ $\Rightarrow$ $-4 = 2 \cdot 0 + b \Rightarrow b = -4 \Rightarrow \boxed{y = 2x - 4}$

b)



① Find two points from the graph:

$(0,3)$ and $(3,0)$

② Let $(x_1, y_1) = (0, 3)$

$(x_2, y_2) = (3, 0)$

and find the slope of line by $\dfrac{y_2 - y_1}{x_2 - x_1}$

$$\text{slope} = \frac{0-3}{3-0} = \frac{-3}{3} = -1$$

③ Find the y-intercept point: $(0,3)$

④ Find the line equation by slope-intercept form

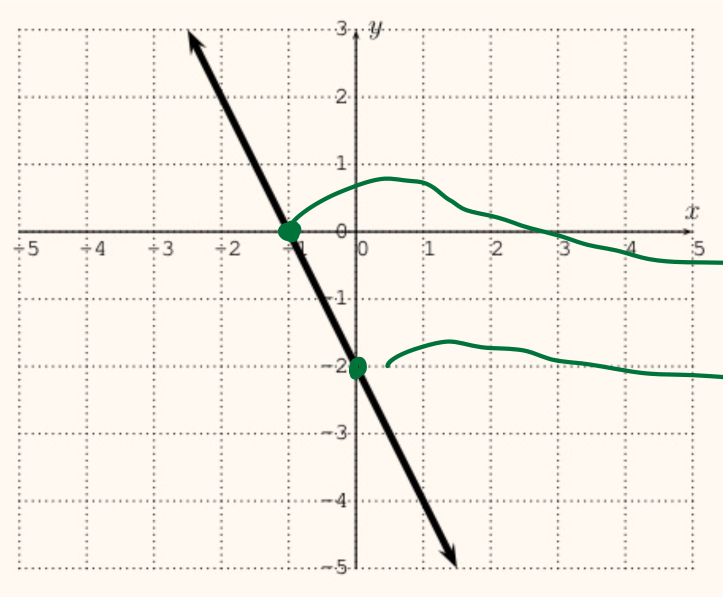
$$y = (\text{slope}) \cdot x + b$$

By ② and ③, $y = -1 \cdot x + b$ and y-intercept is $(0,3)$

$y = -x + b \xrightarrow{\text{plug } (0,3) \text{ into}} 3 = -1 \cdot (0) + b \Rightarrow b = 3$

The line equation is $\boxed{y = -x + 3}$

c)



① Find two points from the graph

$(-1, 0)$ and $(0, -2)$

② Let $(x_1, y_1) = (-1, 0)$
$(x_2, y_2) = (0, -2)$.
and find the slope of the line by $\frac{y_2 - y_1}{x_2 - x_1}$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-0}{0-(-1)} = \frac{-2}{1} = -2$$

③ Find the y- intercept point: $(0, -2)$

④ Find the line equation by slope - intercept form

$$y = (\text{slope}) \cdot x + b$$

By ② and ③, $y = -2x + b$ and y-intercept is $(0, -2)$.

plug $(0, -2)$ into $y = -2x + b$ $\Longrightarrow$ $-2 = -2 \cdot 0 + b \Rightarrow b = -2$

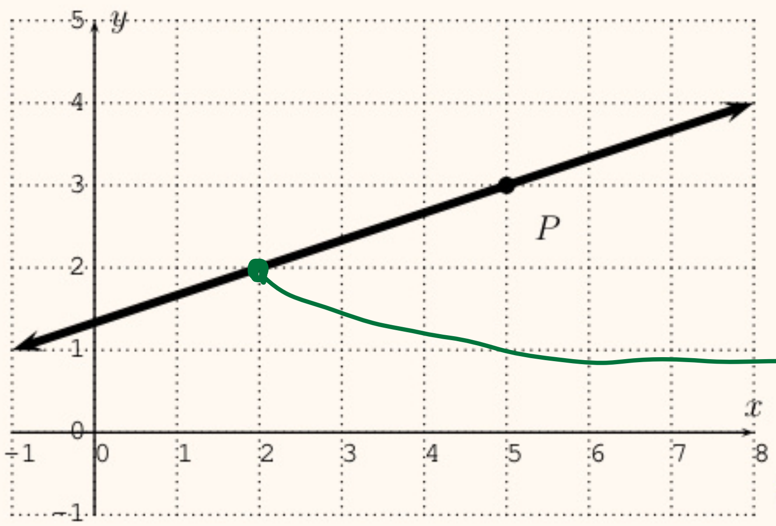
The line equation is $\boxed{y = -2x - 2}$

Exercise 3.2

Find the equation of the line in point-slope form (3.3) using the indicated point P.

Sol

a)



① Name the Point P.

P is $(5, 3)$

② Find the other point from graph

$(2,2)$

③ Let $(x_1, y_1) = (2,2)$
$(x_2, y_2) = (5,3)$. Find the slope $= \dfrac{y_2 - y_1}{x_2 - y_1}$

$$\text{slope} = \frac{3-2}{5-2} = \frac{1}{3}$$

④ Find the equation by point-slope form with $P(5,3)$

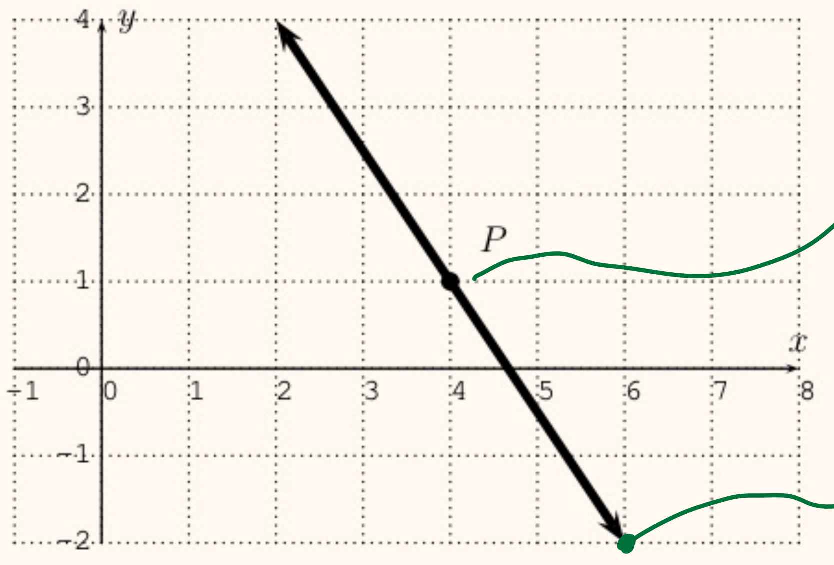
$$\text{slope} = \frac{y-3}{x-5}$$

By ③,
The equation is $\dfrac{1}{3} = \dfrac{y-3}{x-5}$

$$\Rightarrow y-3 = \frac{1}{3}(x-5)$$

$$\Rightarrow \boxed{y = \frac{1}{3}(x-5)+3}$$

b)



① Name the point P

P is $(4, 1)$

② Find the other point from the graph

$(6, -2)$

③ Let $(x_1, y_1) = (4, 1)$, $(x_2, y_2) = (6, -2)$. Find the slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2-1}{6-4} = \frac{-3}{2}$$

④ Find the line equation by point-slope form with $P(4, 1)$:

$$\text{slope} = \frac{y-1}{x-4}$$

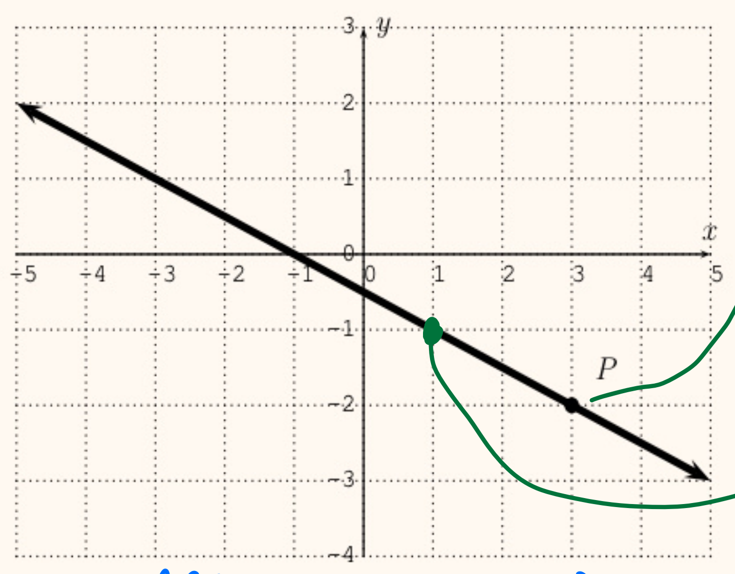
By ③, the equation is

$$-\frac{3}{2} = \frac{y-1}{x-4}$$

$$\Rightarrow y-1 = -\frac{3}{2}(x-4)$$

$$\Rightarrow \boxed{y = -\frac{3}{2}(x-4) + 1}$$

c)



① Name the point P

P is $(3, -2)$

② Find the point from the graph

$(1, -1)$

③ Let $(x_1, y_1) = (1, -1)$, $(x_2, y_2) = (3, -2)$. Find the slope $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$$\text{slope} = \frac{y_2 - y_1}{x_1 - x_2} = \frac{-2-(-1)}{3-1} = \frac{-2+1}{2} = \frac{-1}{2}$$

④ Find the line equation by point-slope form with $P(3, -2)$

$$\text{slope} = \frac{y-(-2)}{x-3}$$

By ③, the line equation is

$$-\frac{1}{2} = \frac{y+2}{x-3}$$

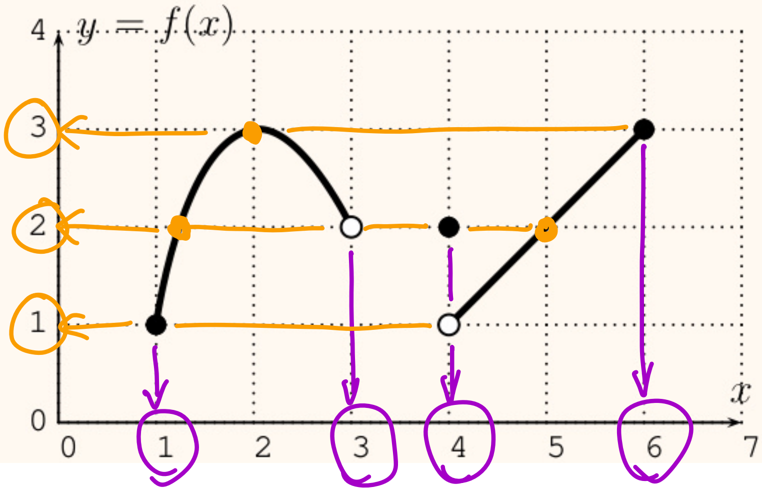
$$\Rightarrow y+2 = -\frac{1}{2}(x-3)$$

$$\Rightarrow \boxed{y = -\frac{1}{2}(x-3) - 2}$$

Exercise 3.3

Below are three graphs for the functions f, g, and h.

function f :



a) Find the domain and range of f.

Sol • For domain of f , we have

① Find the special inputs x that need to discuss

$x=1$ (min. of input, a solid circle)

$x=3$ (an empty circle)

$x=4$ (an empty circle and a solid circle)

$x=6$ (max. of input, a solid circle)

② Discussion of the special points from ①

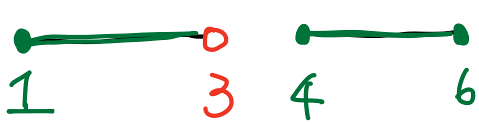
$x=1 \to$ solid point included in domain

$x=3 \to$ empty point excluded in domain

$x=4 \to$ there is a solid point included in domain

$x=6 \to$ solid point included in domain

③ Domain:



$\Rightarrow \boxed{[1,3) \cup [4,6]}$

• For range of f , we have

① Find the special outputs y that need to discuss

$y=1$ (min. of output, a solid circle and an empty circle)

$y=2$ (3 solid circles and an empty circle)

$y=3$ (2 solid circles , max of output)

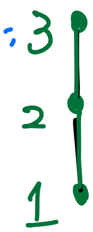
② Discussion of the special points from ①

$y=1 \rightarrow$ it has a solid circle   included in range

$y=2 \rightarrow$ it has 3 solid circles   included in range

$y=3 \rightarrow$ it has 2 solid circles   included in range.

③ Range:



$\Rightarrow$ $\boxed{\text{Range is } [1,3]}$

d) $f(1)$ e) $f(2)$ f) $f(3)$ g) $f(4)$

<u>Sol</u>

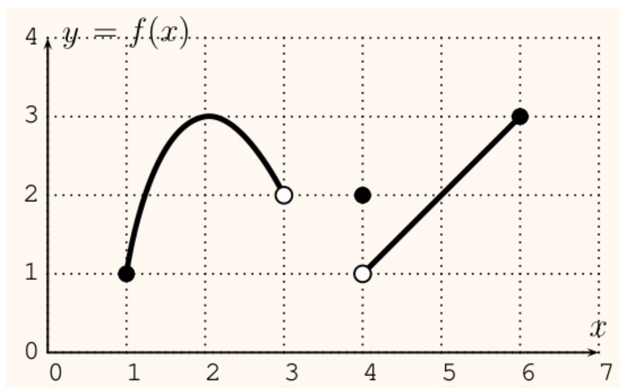
x=1 (input) and ask for output

d) $f(1) = 1$

e) $f(2) = 3$

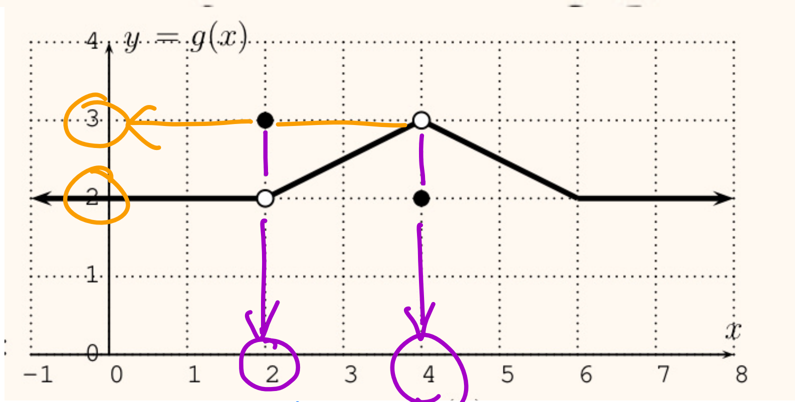
f) $f(3) =$ undefined

g) $f(4) = 2$



b) Find the domain and range of g.

Sol



- For the domain of g, we have

① Find the special inputs x that need to discuss

Arrow to the left: $-\infty$ is the "min."

$X=2$ (an empty circle and a solid circle)

$X=4$ (a solid circle and an empty circle)

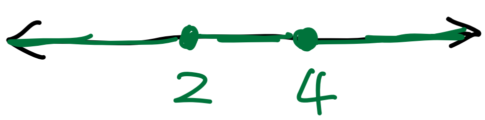
Arrow to the right: ∞ is the "max."

② Discussion:

$X=2 \Rightarrow$ it has a solid circle included in domain

$X=4 \Rightarrow$ it has a solid circle included in domain

③ Domain $\Rightarrow$ $\boxed{\text{domain is } (-\infty, \infty)}$



- For the range of g, we have

① Find the special outputs y that need to discuss.

$y=2$ (infinitely many solid circle and an empty circle)

$y=3$ (one solid circle and an empty circle)

② Discussion

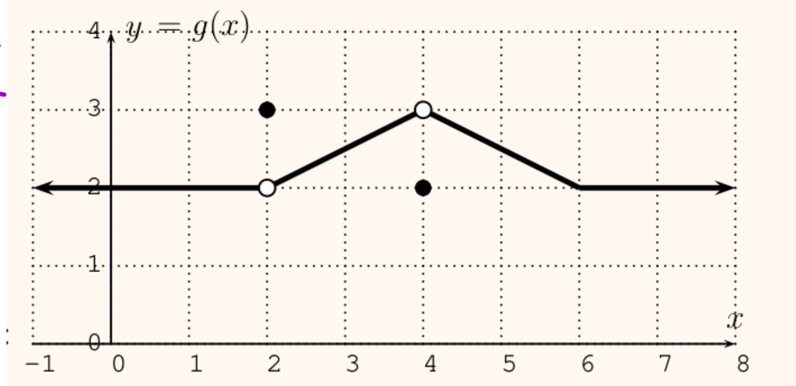
$y=2 \Rightarrow$ it has infinitely many solid circle Included.

$y=3 \Rightarrow$ it has one solid circle Included.

③ Range 3
2
$\Rightarrow$ $\boxed{\text{range is } [2,3]}$

m) $g(2)$ n) $g(3)$ o) $g(4)$ p) $g(6)$ q) $g(13.2)$

Sol. $x=2$ (input) and ask for output



$g(2) = 3$

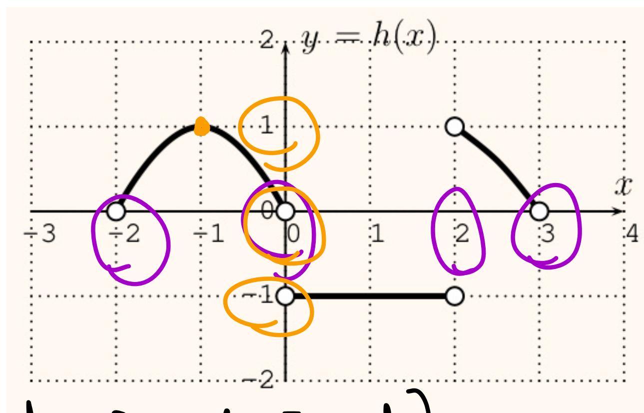
$g(3) = 2.5$

$g(4) = 2$

$g(6) = 2$

$g(13.2) = 2$

c) Find the domain and range of h.



Sol. • For the domain of h, we have

① Find the special inputs x that need to discuss:

$x=-2$ (one empty circle and min. of input)

$x=0$ (two empty circles)

$x=2$ (two empty circles)

$x=3$ (one empty circles and max. of input)

② Discussion:

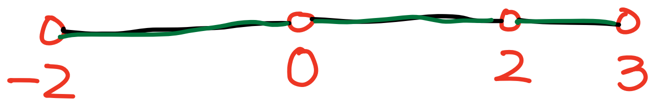
$x=-2 \to$ it has no solid circle — excluded

$x=0 \to$ it has no solid circle — excluded

$x=2 \to$ it has no solid circle — excluded

$x=3 \to$ it has no solid circle — excluded

③ Domain $\Rightarrow$ $\boxed{(-2,0) \cup (0,2) \cup (2,3)}$



- For the range of h, we have

① Find the special outputs y that need to discuss

$y=1$ (max of output and one empty circle, one solid circle)

$y=0$ (3 solid circles)

$y=-1$ (min of output and infinitely many solid points and 2 empty circles)

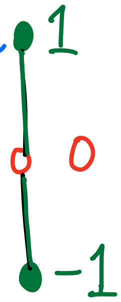
② Discussion

$y=1 \to$ it has a solid circle — Included

$y=0 \to$ it has no solid circle — excluded

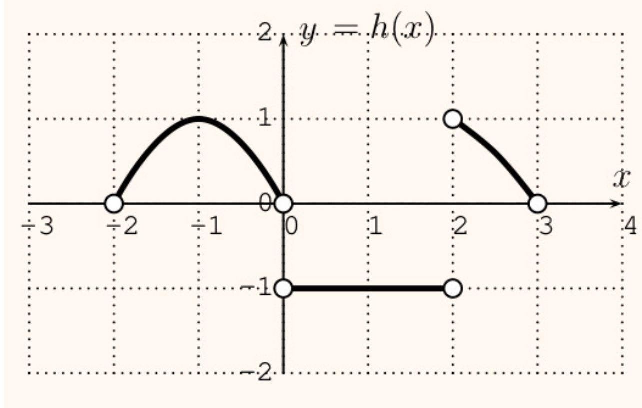
$y=-1 \to$ it has solid circles — included

③ Range



$\Rightarrow$ $\boxed{\text{range is } [-1,0) \cup (0,1]}$

r) $h(-2)$ s) $h(-1)$ t) $h(0)$



Sol

$h(-2) =$ undefined

$h(-1) = 1$

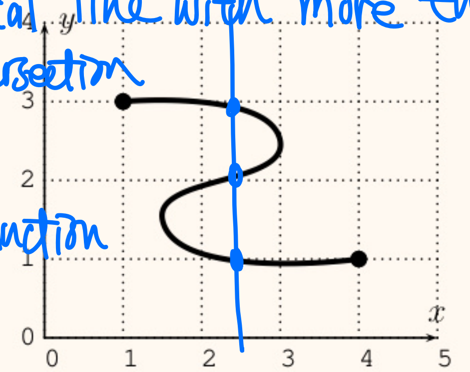
$h(0) =$ undefined.

Exercise 3.4

Use the vertical line test to determine which of the following graphs are the graphs of functions.

a) a vertical line with more than one intersection point

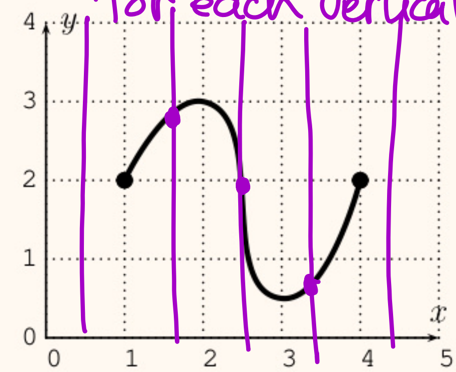
$\Rightarrow$ NOT a function



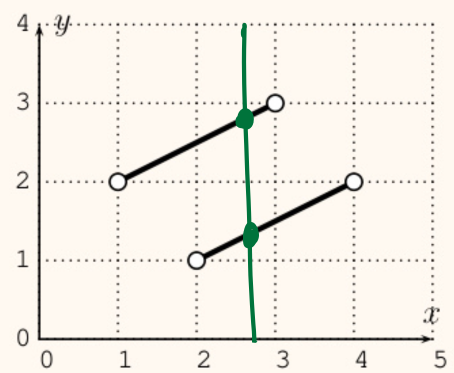
a)

b) At most one intersection point for each vertical line

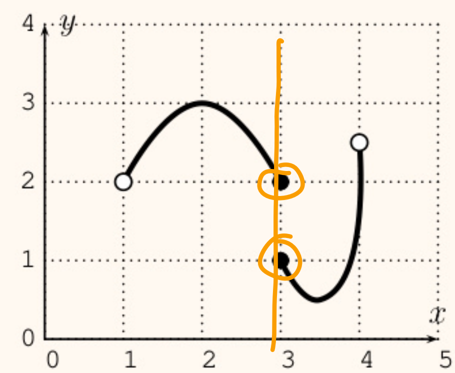
$\Rightarrow$ it's a function



b)



c)



d)

c) a vertical line with more than one intersection point

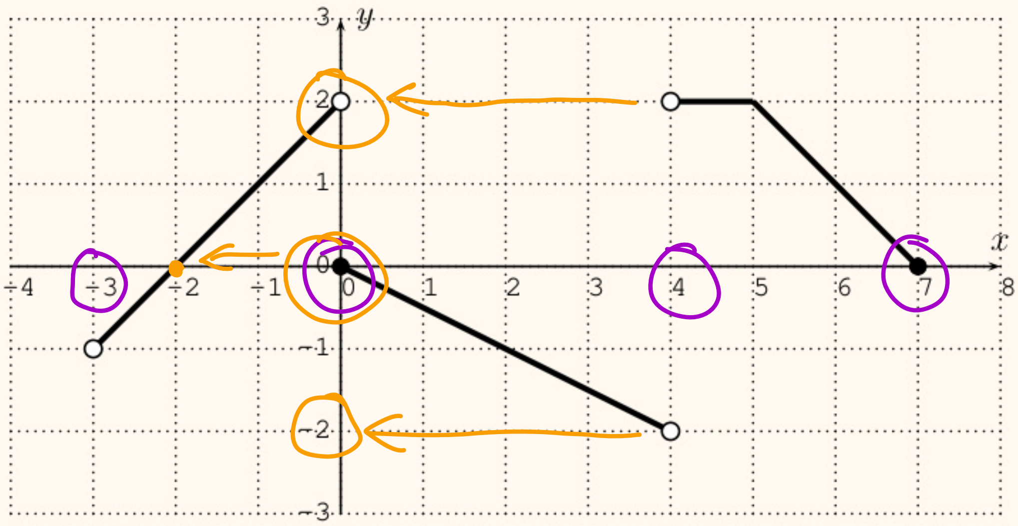
$\Rightarrow$ Not a function

d) a vertical line with more than one intersection point

$\Rightarrow$ NOT a function

Exercise 3.5

Let f be the function given by the following graph.



a) What is the domain of f? b) What is the range of f?

Sol

a) For the domain of f, we have

① Find the special inputs x that need to discuss

$x=-3$ (min. of input and an empty circle)

$x=0$ (an empty circle and a solid circle)

$x=4$ (two empty circles)

$x=7$ (max of input and a solid circle)

② Discussion

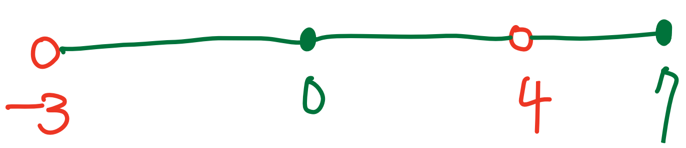
$x=-3 \to$ it has no solid circle $\Rightarrow$ excluded

$x=0 \to$ it has a solid circle $\Rightarrow$ included

$x=4 \to$ it has no solid circle $\Rightarrow$ excluded

$x=7 \to$ it has a solid circle $\Rightarrow$ included

③ Domain



$\Rightarrow$ domain is $\boxed{(-3,4) \cup (4,7]}$

b) For the range of f, we have

① Find the special outputs y that need to discuss.

$y=2$ (two empty circle and infinitely many solid circles and max of output)

$y=0$ (three solid point)

$y=-2$ (min of output and an empty circle)

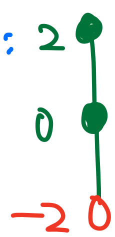
② Discussion:

$y=2 \Rightarrow$ it has solid circles $\longrightarrow$ Included

$y=0 \Rightarrow$ it has solid circles $\longrightarrow$ Included

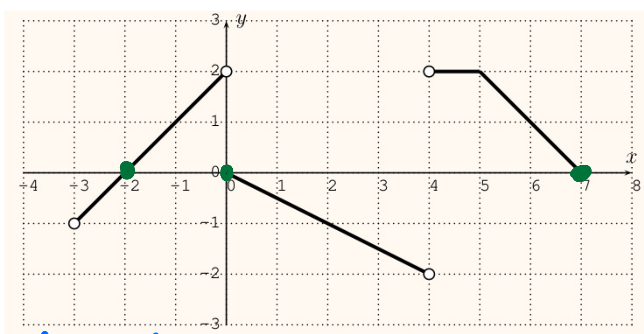
$y=-2 \Rightarrow$ it has no solid circle. $\longrightarrow$ excluded

③ Range:



$\Rightarrow$ $\boxed{\text{the range is } (-2, 2].}$

c) For which x is $f(x) = 0$?
d) For which x is $f(x) = 2$?
e) For which x is $f(x) \le 1$?
f) For which x is $f(x) > 0$?
g) Find $f(2)$ and $f(5)$.
h) Find $f(2) + f(5)$.
i) Find $f(2) + 5$.
j) Find $f(2 + 5)$.

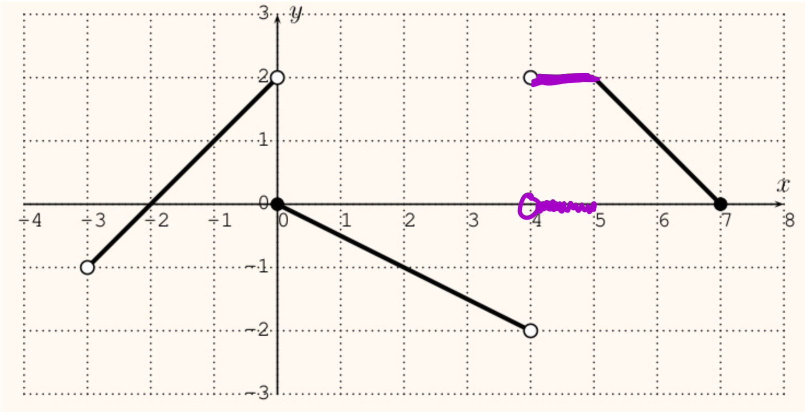


Sol

(c) Find the input(s) x such that the output(s) y is 0 $(f(x)=0)$.

$X=-2$, $X=0$, $X=7$

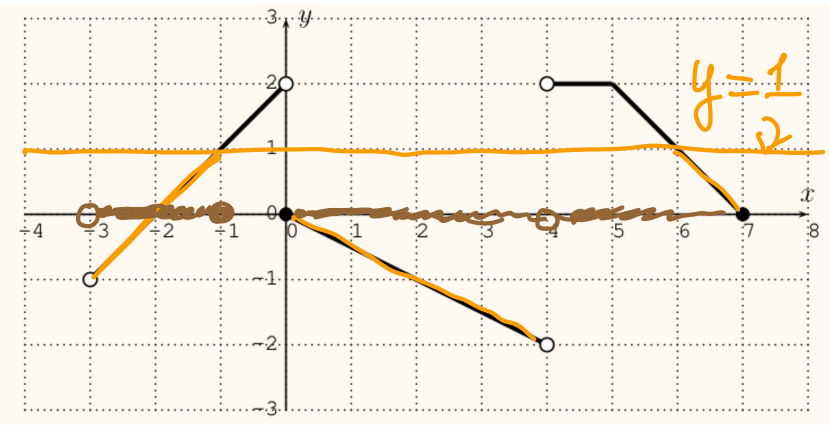
(d) Find the input(s) x such that the outputs y is 2 ($f(x)=2$)

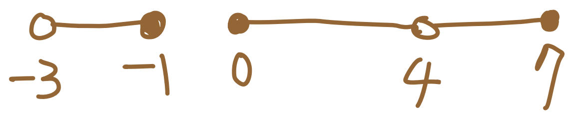


$\boxed{x \in (4,5]}$

(it is not just a point but an interval)

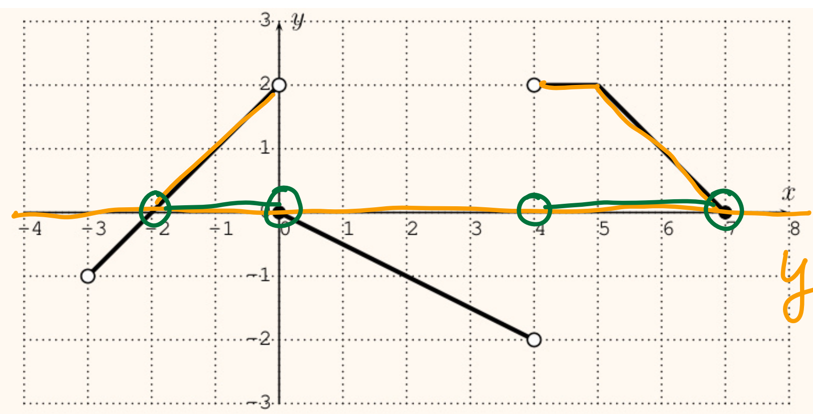
(e) Find the input(s) x such that the output(s) $y \le 1$ ($f(x) \le 1$)



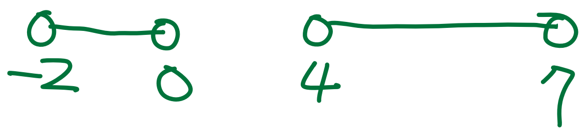


$\boxed{x \in (-3,-1] \cup [0,4) \cup (4,7]}$

(f) Find the input(s) x such that the output(s) $y > 0$ ($f(x) > 0$)



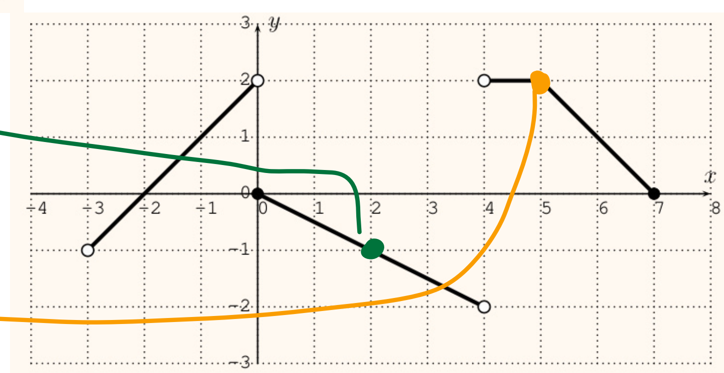
$y=0$



$x \in (-2,0) \cup (4,7)$

(g) $f(2) = -1$

$f(5) = 2$



(h) $f(2)+f(5) = -1 + 2 = 1$

(i) $f(2)+5 = -1+5 = 4$

(j) $f(2+5) = f(7) = 0$