We still need to find a_{333} in the above formula:

$$a_{333} = 15 - (333 - 1) \cdot 4 = 15 - 332 \cdot 4 = 15 - 1328 = -1313$$

This gives a total sum of

$$\sum_{n=1}^{333} a_n = \frac{333}{2} \cdot (15 + (-1313)) = \frac{35}{2} \cdot (-1298) = -216,117.$$

□

24.3   Exercises

Exercise 24.1

Find the first seven terms of the sequence.

a) $a_n = 3n$ b) $a_n = 5n + 3$ c) $a_n = n^2 + 2$
d) $a_n = n$ e) $a_n = (-1)^{n+1}$ f) $a_n = \frac{\sqrt{n+1}}{n}$
g) $a_k = 10^k$ h) $a_i = 5 + (-1)^i$ i) $a_n = \sin(\frac{\pi}{2} \cdot n)$

Exercise 24.2

Find the first six terms of the sequence.

a) $a_1 = 5,$ $a_n = a_{n-1} + 3$ for $n \geq 2$
b) $a_1 = 7,$ $a_n = 10 \cdot a_{n-1}$ for $n \geq 2$
c) $a_1 = 1,$ $a_n = 2 \cdot a_{n-1} + 1$ for $n \geq 2$
d) $a_1 = 6,$ $a_2 = 4,$ $a_n = a_{n-1} - a_{n-2}$ for $n \geq 3$

Exercise 24.3

Find the value of the series.

a) $\sum_{n=1}^{4} a_n,$ where $a_n = 5n$ b) $\sum_{k=1}^{5} a_k,$ where $a_k = k$
c) $\sum_{i=1}^{4} a_i,$ where $a_n = n^2$ d) $\sum_{n=1}^{6} (n - 4)$
e) $\sum_{k=1}^{3} (k^2 + 4k - 4)$ f) $\sum_{j=1}^{4} \frac{1}{j+1}$

Exercise 24.4

Is the sequence below part of an arithmetic sequence? If it is part of an arithmetic sequence, find the formula for the nth term a_n in the form $a_n = a_1 + (n-1) \cdot d$.

a) $5, 8, 11, 14, 17, \ldots$

b) $-10, -7, -4, -1, 2, \ldots$

c) $-1, 1, -1, 1, -1, 1, \ldots$

d) $18, 164, 310, 474, \ldots$

e) $73.4, 51.7, 30, \ldots$

f) $9, 3, -3, -8, -14, \ldots$

g) $4, 4, 4, 4, 4, \ldots$

h) $-2.72, -2.82, -2.92, -3.02, -3.12, \ldots$

i) $\sqrt{2}, \sqrt{5}, \sqrt{8}, \sqrt{11}, \ldots$

j) $\frac{-3}{5}, \frac{-1}{10}, \frac{2}{5}, \ldots$

k) $a_n = 4 + 5 \cdot n$

l) $a_j = 2 \cdot j - 5$

m) $a_n = n^2 + 8n + 15$

n) $a_k = 9 \cdot (k+5) + 7k - 1$

Exercise 24.5

Determine the general nth term a_n of an arithmetic sequence $\{a_n\}$ with the data given below.

a) $d = 4$, and $a_8 = 57$

b) $d = -3$, and $a_{99} = -70$

c) $a_1 = 14$, and $a_7 = -16$

d) $a_1 = -80$, and $a_5 = 224$

e) $a_3 = 10$, and $a_{14} = -23$

f) $a_{20} = 2$, and $a_{60} = 32$

Exercise 24.6

Determine the value of the indicated term of the given arithmetic sequence.

a) if $a_1 = 8$, and $a_{15} = 92$, find a_{19}

b) if $d = -2$, and $a_3 = 31$, find a_{81}

c) if $a_1 = 0$, and $a_{17} = -102$, find a_{73}

d) if $a_7 = 128$, and $a_{37} = 38$, find a_{26}

Exercise 24.7

Determine the sum of the arithmetic sequence.

a) Find the sum $a_1 + \cdots + a_{48}$ for the arithmetic sequence $a_n = 4n + 7$.

b) Find the sum $\sum_{n=1}^{21} a_n$ for the arithmetic sequence $a_n = 2 - 5n$.

c) Find the sum: $\displaystyle\sum_{n=1}^{99}(10\cdot n+1)$

d) Find the sum: $\displaystyle\sum_{n=1}^{200}(-9-n)$

e) Find the sum of the first 100 terms of the arithmetic sequence:

$$2, 4, 6, 8, 10, 12, \ldots$$

f) Find the sum of the first 83 terms of the arithmetic sequence:

$$25, 21, 17, 13, 9, 5, \ldots$$

g) Find the sum of the first 75 terms of the arithmetic sequence:

$$2012, 2002, 1992, 1982, \ldots$$

h) Find the sum of the first 16 terms of the arithmetic sequence:

$$-11, -6, -1, \ldots$$

i) Find the sum of the first 99 terms of the arithmetic sequence:

$$-8, -8.2, -8.4, -8.6, -8.8, -9, -9.2, \ldots$$

j) Find the sum

$$7 + 8 + 9 + 10 + \cdots + 776 + 777$$

k) Find the sum of the first 40 terms of the arithmetic sequence:

$$5, 5, 5, 5, 5, \ldots$$