the scalar product. These operations have to satisfy the following properties.

Associativity:	$(\vec{u}+\vec{v})+\vec{w}=\vec{u}+(\vec{v}+\vec{w})$
Commutativity:	$\vec{v}+\vec{w}=\vec{w}+\vec{v}$
Zero element:	there is a vector $\vec{o}$ such that $\vec{o}+\vec{v}=\vec{v}$ and $\vec{v}+\vec{o}=\vec{v}$ for every vector $\vec{v}$
Negative element:	for every $\vec{v}$ there is a vector $-\vec{v}$ such that $\vec{v}+(-\vec{v})=\vec{o}$ and $(-\vec{v})+\vec{v}=\vec{o}$
Distributivity (1):	$r\cdot(\vec{v}+\vec{w})=r\cdot\vec{v}+r\cdot\vec{w}$
Distributivity (2):	$(r+s)\cdot\vec{v}=r\cdot\vec{v}+s\cdot\vec{v}$
Scalar compatibility:	$(r\cdot s)\cdot\vec{v}=r\cdot(s\cdot\vec{v})$
Identity:	$1\cdot\vec{v}=\vec{v}$

An important example of a vector space is the 2-dimensional plane $V=\mathbb{R}^2$ as it was discussed in this chapter. A thorough introduction to this topic will be provided in a course in linear algebra.

22.3 Exercises

Exercise 22.1

Graph the vectors in the plane.

a) $\overrightarrow{PQ}$ with $P(2,1)$ and $Q(4,7)$
b) $\overrightarrow{PQ}$ with $P(-3,3)$ and $Q(-5,-4)$
c) $\overrightarrow{PQ}$ with $P(0,-4)$ and $Q(6,0)$
d) $\langle -2,4\rangle$
e) $\langle -3,-3\rangle$
f) $\langle 5,5\sqrt{2}\rangle$

Exercise 22.2

Find the magnitude and direction angle of the vector.

a) $\langle 6,8\rangle$
b) $\langle -2,5\rangle$
c) $\langle -4,-4\rangle$
d) $\langle 3,-3\rangle$
e) $\langle 2,-2\rangle$
f) $\langle 4\sqrt{3},4\rangle$
g) $\langle -\sqrt{3},-1\rangle$
h) $\langle -4,4\sqrt{3}\rangle$
i) $\langle -2\sqrt{3},-2\rangle$

j) $\overrightarrow{PQ}$, where $P(3,1)$ and $Q(7,4)$
k) $\overrightarrow{PQ}$, where $P(4,-2)$ and $Q(-5,7)$

Exercise 22.3

Perform the operation on the vectors.

a) $5 \cdot \langle 3, 2\rangle$
b) $2 \cdot \langle -1, 4\rangle$
c) $(-10) \cdot \langle -\frac{3}{2}, -\frac{7}{5}\rangle$
d) $\langle 2, 3\rangle + \langle 6, 1\rangle$
e) $\langle 5, -4\rangle - \langle -8, -9\rangle$
f) $3 \cdot \langle 5, 3\rangle + 4 \cdot \langle 2, 8\rangle$
g) $(-2)\langle -5, -4\rangle - 6\langle -1, -2\rangle$
h) $\frac{2}{3}\langle -3, 6\rangle - \frac{7}{5}\langle 10, -15\rangle$
i) $\sqrt{2} \cdot \langle \frac{\sqrt{8}}{6}, \frac{-5\sqrt{2}}{12}\rangle - 2\langle \frac{2}{3}, \frac{5}{3}\rangle$
j) $6\vec{i} - 4\vec{j}$
k) $-5\vec{i} + \vec{j} + 3\vec{i}$
l) $3 \cdot \langle -4, 2\rangle - 8\vec{j} + 12\vec{i}$

m) find $4\vec{v} + 7\vec{w}$ for $\vec{v} = \langle 2, 3\rangle$ and $\vec{w} = \langle 5, 1\sqrt{3}\rangle$
n) find $\vec{v} - 2\vec{w}$ for $\vec{v} = \langle -11, -6\rangle$ and $\vec{w} = \langle -3, 2\rangle$
o) find $3\vec{v} - \vec{w}$ for $\vec{v} = -4\vec{i} + 7\vec{j}$ and $\vec{w} = 6\vec{i} + \vec{j}$
p) find $-\vec{v} - \sqrt{5}\vec{w}$ for $\vec{v} = 5\vec{j}$ and $\vec{w} = -8\vec{i} + \sqrt{5}\vec{j}$

Exercise 22.4

Find a unit vector in the direction of the given vector.

a) $\langle 8, -6\rangle$
b) $\langle -3, -\sqrt{7}\rangle$
c) $\langle 9, 2\rangle$
d) $\langle -\sqrt{5}, \sqrt{31}\rangle$
e) $\langle 5\sqrt{2}, 3\sqrt{10}\rangle$
f) $\langle 0, -\frac{3}{5}\rangle$

Exercise 22.5

Find the approximate magnitude and direction angle of sum $\vec{v} = \vec{v_1} + \vec{v_2}$ of the given vectors $\vec{v_1}$ and $\vec{v_2}$ (see Example 22.18).

a) $||\vec{v_1}|| = 6$, and $\theta_1 = 60°$, and
$||\vec{v_2}|| = 2$, and $\theta_2 = 180°$

b) $||\vec{v_1}|| = 3.7$, and $\theta_1 = 92°$, and
$||\vec{v_2}|| = 2.2$, and $\theta_2 = 253°$

c) $||\vec{v_1}|| = 8$, and $\theta_1 = \frac{3\pi}{4}$, and
$||\vec{v_2}|| = 8\sqrt{2}$, and $\theta_2 = \frac{3\pi}{2}$