21.3 Exercises

Exercise 21.1

Write the expression as one of the six trigonometric functions.

a) $\cos(x) \cdot \tan(x)$ b) $\sec(x) \cdot \cot(x)$ c) $\frac{\csc(x)}{\sec(x)}$

d) $\tan(x) \cdot \frac{\cot(x)}{\sin(x)}$ e) $\frac{\cot(x)}{\csc(x)}$ f) $\frac{\sin(x)}{\cot(x)} \cdot \csc^2(x)$

Exercise 21.2

Determine if the identity is true or false. If the identity is true, then give an argument for why it is true.

a) $\cos(x) \cdot \csc(x) = \sin(x) \cdot \sec(x)$

b) $\frac{\sin(x)}{\cot(x)} = \frac{\tan(x)}{\csc(x)}$

c) $\frac{\csc(x)}{\sin(x)} = \frac{\cot(x)}{\tan(x)}$

d) $\sin(x) \cdot \cos(x) \cdot \csc^2(x) = \frac{\csc(x)}{\sec(x)}$

Exercise 21.3

Simplify the expression as much as possible.

a) $\frac{\cos^2(x)-1}{\sin(x)}$ b) $\frac{1-\sin^2(x)}{\cot(x)}$

c) $1 + \frac{\cos^2(x)}{\sin^2(x)}$ d) $\frac{\tan^2(x)}{\sec^2(x)} - 1$

e) $\cos(x) + \frac{\sin^2(x)}{\cos(x)}$ f) $\sec(x) - \frac{\tan^2(x)}{\sec(x)}$

g) $(1 + \sin(x)) \cdot (1 - \sin(x))$ h) $(1 - \sec(x)) \cdot (1 + \sec(x))$

i) $(\csc(x) - 1) \cdot (\csc(x) + 1)$ j) $\frac{\sec(x)}{\tan(x)} - \frac{\tan(x)}{\sec(x)}$

k) $\cos^4(x) - \sin^4(x)$ l) $\tan^4(x) - \sec^4(x)$

Exercise 21.4

Determine whether the identity is true or false. If the identity is true, then give an argument for why it is true.

a) $\sin(x) - \sin(x)\cos^2(x) = \sin^3(x)$
b) $\cot^2(x) - \csc^2(x) = \tan^2(x) - \sec^2(x)$
c) $\tan^2(x) + \sec^2(x) = 1$
d) $\sin^3(x) - \sin(x) = -\sin(x) \cdot \cos^2(x)$
e) $\sin(x) \cdot (\cos(x) - \sin(x)) = \cos^2(x)$
f) $(\sin(x) - \cos(x))^2 = 1 - 2\sin(x)\cos(x)$

Exercise 21.5

Simplify the expression as much as possible.

a) $\sin(x + \pi)$ b) $\tan(\pi - x)$ c) $\cot(x + \frac{\pi}{2})$ d) $\cos(x + \frac{3\pi}{2})$

Exercise 21.6

Find the exact values of the trigonometric functions of $\frac{\alpha}{2}$ and of 2α by using the half-angle and double-angle formulas.

a) $\sin(\alpha) = \frac{4}{5}$, and α in quadrant I
b) $\cos(\alpha) = \frac{7}{13}$, and α in quadrant IV
c) $\sin(\alpha) = \frac{-3}{5}$, and α in quadrant III
d) $\tan(\alpha) = \frac{4}{3}$, and α in quadrant III
e) $\tan(\alpha) = \frac{-5}{12}$, and α in quadrant II
f) $\cos(\alpha) = \frac{-2}{3}$, and α in quadrant II