Mat 1375 HW1

Exercise 1.1

Give examples of numbers that are

a) natural numbers → $0, 1, 2, 3, 4, \cdots$

b) integers → $\cdots, -3, -2, -1, 0, 1, 2, 3, \cdots$

c) integers but not natural numbers → $-5, -4, -3, -2, -1$

d) rational numbers → $\frac{1}{2}, \frac{3}{5}, \frac{-7}{4}, \frac{-3}{1}, 10.$

e) real numbers → $1, \frac{3}{5}, \sqrt{2}, \pi, e$

f) rational numbers but not integers → $\frac{1}{2}, \frac{3}{5}, -\frac{7}{4}, -\frac{3}{2}$

Exercise 1.2

Which of the following numbers are natural numbers, integers, rational numbers, or real numbers? Which of these numbers are irrational?

a) $\frac{7}{3}$ b) -5 c) 0 d) $17,000$ e) $\frac{12}{4}$ f) $\sqrt{7}$ g) $\sqrt{25}$

a) rational number , real number

b) integer, rational number, real number

c) integer , (natural number), rational number , real number

d) natural number, integer, rational number, real number

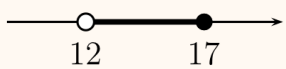
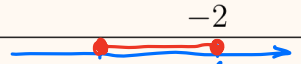
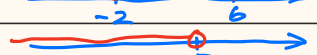
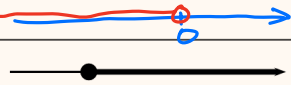
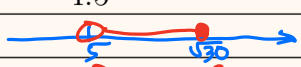
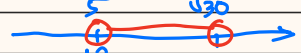
e) natural number, integer, rational number, real number

f) real number , irrational number

g) natural number, integer, rational number, real number

Exercise 1.3

Complete the table.

Inequality notation	Number line	Interval notation
$2 \le x < 5$		$[2, 5)$
$x \le 3$		$(-\infty, 3]$
$12 < x \le 17$		$(12, 17]$
$x < -2$		$(-\infty, -2)$
$-2 \le x \le 6$		$[-2, 6]$
$x < 0$		$(-\infty, 0)$
$x \ge 4.5$		$[4.5, \infty)$
$5 < x \le \sqrt{30}$		$(5, \sqrt{30}]$
$\frac{13}{7} < x < \pi$		$(\frac{13}{7}, \pi)$

Exercise 1.5

In a store, every item that is for sale has a price.

a) Does the assignment which assigns to an item its price constitute a function (in the sense of Definition 1.8 on page 6)?

b) Does the assignment which assigns to a given price all items with this price constitute a function?

c) In the case where the assignment is a function, what is the domain?

d) In the case where the assignment is a function, what is the range?

a) Yes, an item (input) only gets one price (output)
So "assigning to an item its price" is a function

b) NO, there might be the price assigns to two different items.
(input) (outputs)

c) In (a), domain is items.

d) In (a), range is prices.

Exercise 1.6

A bank offers wealthy customers a certain amount of interest if they keep more than 1 million dollars in their account. The amount is described in the following table.

dollar amount x in the account	interest amount
$x \leq \$1,000,000$	$\$0$
$\$1,000,000 < x \leq \$10,000,000$	2% of x
$\$10,000,000 < x$	1% of x

a) Justify that the assignment cash amount to interest defines a function.

b) Find the interest for an amount of:

i) $\$50,000$ ii) $\$5,000,000$ iii) $\$1,000,000$
iv) $\$30,000,000$ v) $\$10,000,000$ vi) $\$2,000,000$

a) $\text{Interest} = \begin{cases} 0 & , X \leq 1\text{Million} \\ 0{,}02X & , 1\text{Million} < X \leq 10\text{ Millions} \\ 0{,}01X & , X > 10\text{ Millions} \end{cases}$

Since each x (or input) only gets one interest (output), so it is a function.

b) i) $\$50{,}000 \leq 1M \Rightarrow$ interest $= 0$

ii) $1M < \$5{,}000{,}000 \leq 10M \Rightarrow$ interest $= 5M \cdot 0.02 = 0.1M$

iii) $\$1M \leq 1M \Rightarrow$ interest $= 0$

iv) $\$30M > 10M \Rightarrow$ interest $= 30M \cdot 0.01 = 0.3M$

v) $1M < \$10M \leq 10M \Rightarrow$ interest $= 10M \cdot 0.02 = 0.2M$

vi) $1M < \$2M \leq 10M \Rightarrow$ interest $= 2M \cdot 0.02 = 0.04M$

Exercise 1.7

Find a formula for a function describing the given inputs and outputs.

a) *input*: the radius of a circle r
output: the circumference of the circle C
$\Rightarrow C = f(r) = 2\pi r$

b) *input*: the side length in an equilateral triangle l
output: the perimeter of the triangle p
$\Rightarrow p = f(l) = 3l$

c) *input*: one side length of a rectangle, with other side length being 3 l
output: the perimeter of the rectangle p
$\Rightarrow p = f(l) = 2l + 2 \cdot 3 = 2l + 6$

d) *input*: the side length of a cube l
output: the volume of the cube V
$\Rightarrow V = f(l) = l^3$.