$$\begin{aligned} & \xrightarrow{(\times \frac{5730}{\ln(0.5)})} && \frac{5730}{\ln(0.5)} \cdot \ln(0.86) = t \\ & \Longrightarrow && t \approx 1247 \end{aligned}$$

Therefore, the tree died approximately 1247 years ago.

b) Since 41.3% of the carbon-14 is gone, $100\% - 41.3\% = 58.7\%$ remains. Using $y = c \cdot \left(\frac{1}{2}\right)^{\frac{t}{h}}$ with $y = 58.7\% \cdot c$ and $h = 5730$, we obtain

$$\begin{aligned} 0.587 \cdot c = c \cdot \left(\frac{1}{2}\right)^{\frac{t}{5730}} & \xrightarrow{(\div c)} && 0.587 = \left(\frac{1}{2}\right)^{\frac{t}{5730}} \\ & \xrightarrow{\text{(apply ln)}} && \ln(0.587) = \ln\left(0.5^{\frac{t}{5730}}\right) \\ & \Longrightarrow && \ln(0.587) = \frac{t}{5730} \cdot \ln(0.5) \\ & \xrightarrow{(\times \frac{5730}{\ln(0.5)})} && \frac{5730}{\ln(0.5)} \cdot \ln(0.587) = t \\ & \Longrightarrow && t \approx 4404 \end{aligned}$$

The animal died 4404 years ago.

□

16.3 Exercises

Exercise 16.1

An investment of $\$5000$ was locked in for 30 years. According to the agreed-upon conditions, the investment will be worth $\$5000 \cdot 1.08^t$ after t years.

a) How much is the investment worth after 5 years?
b) After how many years will the investment be worth $\$20,000$?

Exercise 16.2

Determine the final amount in a savings account under the given conditions.

a)	$700,	compounded quarterly,	at 3%,	for 7 years
b)	$1400,	compounded annually,	at 2.25%,	for 5 years
c)	$1400,	compounded continuously,	at 2.25%,	for 5 years
d)	$500,	compounded monthly,	at 3.99%,	for 2 years
e)	$5000,	compounded continuously,	at 7.4%,	for 3 years
f)	$1600,	compounded daily,	at 3.333%,	for 1 year
g)	$750,	compounded semi-annually,	at 4.9%,	for 4 years

Exercise 16.3

a) Find the amount P that needs to be invested at a rate of 5% compounded quarterly for 6 years to give a final amount of $\$2000$.

b) Find the present value P of a future amount of $A = \$3500$ invested at 6% compounded annually for 3 years.

c) Find the present value P of a future amount of $\$1000$ invested at a rate of 4.9% compounded continuously for 7 years.

d) At what rate do we have to invest $\$1900$ for 4 years compounded monthly to obtain a final amount of $\$2250$?

e) At what rate do we have to invest $\$1300$ for 10 years compounded continuously to obtain a final amount of $\$2000$?

f) For how long do we have to invest $\$3400$ at a rate of 5.125% compounded annually to obtain a final amount of $\$3700$?

g) For how long do we have to invest $\$1000$ at a rate of 2.5% compounded continuously to obtain a final amount of $\$1100$?

h) How long do you have to invest a principal at a rate of 6.75% compounded daily until the investment doubles its value?

i) A certain amount of money has tripled its value while being in a savings account that has an interest rate of 8% compounded continuously. For how long was the money in the savings account?

Exercise 16.4

An unstable element decays at a rate of 5.9% per minute. If 40mg of this element has been produced, how long will it take until 2mg of the element are left? Round your answer to the nearest thousandth.

Exercise 16.5

A substance decays radioactively with a half-life of 232.5 days. How much of 6.8 grams of this substance is left after 1 year?

Exercise 16.6

Fermium-252 decays in 10 minutes to 76.1% of its original mass. Find the half-life of fermium-252.

Exercise 16.7

How long do you have to wait until 15mg of beryllium-7 have decayed to 4mg if the half-life of beryllium-7 is 53.12 days?

Exercise 16.8

If Pharaoh Ramses II died in the year 1213 BC, then what percent of the carbon-14 was left in the mummy of Ramses II in the year 2000?

Exercise 16.9

In order to determine the age of a piece of wood, the amount of carbon-14 was measured. It was determined that the wood had lost 33.1% of its carbon-14. How old is this piece of wood?

Exercise 16.10

Archaeologists uncovered a bone at an ancient resting ground. It was determined that 62% of the carbon-14 was left in the bone. How old is the bone?