$$\begin{aligned} &\Longrightarrow && \ln(2) = \ln(e^{0.098 \cdot x}) \\ &\Longrightarrow && \ln(2) = 0.098 \cdot x \ln(e) \\ &\Longrightarrow && x = \frac{\ln(2)}{0.098} \approx 7.07 \end{aligned}$$

Therefore, it took about 7.07 weeks until the number of flu cases doubled.

b) Since the number of flu cases was decreasing, the rate of growth is negative, $r = -15\% = -0.15$ per week, so that we have an exponential function with base $b = e^r = e^{-0.15}$. To reach a quarter of its initial number of flu cases, we set $f(x) = c \cdot e^{-0.15 \cdot x}$ equal to $\frac{1}{4}c$.

$$\begin{aligned} \frac{1}{4}c = c \cdot e^{-0.15 \cdot x} \quad &\overset{(\div c)}{\Longrightarrow} \quad \frac{1}{4} = e^{-0.15 \cdot x} \\ &\Longrightarrow \quad \ln(\frac{1}{4}) = -0.15 \cdot x \cdot \ln(e) \\ &\Longrightarrow \quad x = \frac{\ln(\frac{1}{4})}{-0.15} \approx 9.24 \end{aligned}$$

It therefore took about 9.24 weeks until the number of flu cases decreased to a quarter.

□

15.3 Exercises

Exercise 15.1

Solve for x without using a calculator.

a)	$6^{x-2} = 36$	b)	$2^{3x-8} = 16$
c)	$10^{5-x} = 0.0001$	d)	$5^{5x+7} = \frac{1}{125}$
e)	$2^x = 64^{x+1}$	f)	$4^{x+3} = 32^x$
g)	$13^{4+2x} = 1$	h)	$3^{x+2} = 27^{x-3}$
i)	$25^{7x-4} = 5^{2-3x}$	j)	$9^{5+3x} = 27^{8-2x}$

Exercise 15.2

Solve for x. First find the exact answer as an expression involving logarithms. Then approximate the answer to the nearest hundredth using a calculator.

a) $4^x = 57$
b) $9^{x-2} = 7$
c) $2^{x+1} = 31$
d) $3.8^{2x+7} = 63$
e) $5^{x+5} = 8^x$
f) $3^{x+2} = 0.4^x$
g) $1.02^{2x-9} = 4.35^x$
h) $4^{x+1} = 5^{x+2}$
i) $9^{3-x} = 4^{x-6}$
j) $2.4^{7-2x} = 3.8^{3x+4}$
k) $4^{9x-2} = 9^{2x-4}$
l) $1.95^{-3x-4} = 1.2^{4-7x}$

Exercise 15.3

Assuming that $f(x) = c \cdot b^x$ is an exponential function, find the constants c and b from the given conditions.

a) $f(0) = 4$, $f(1) = 12$
b) $f(0) = 5$, $f(3) = 40$
c) $f(0) = 3200$, $f(6) = 0.0032$
d) $f(3) = 12$, $f(5) = 48$
e) $f(-1) = 4$, $f(2) = 500$
f) $f(2) = 3$, $f(4) = 15$

Exercise 15.4

The number of downloads of a certain software application was 8.4 million in the year 2017 and 13.6 million in the year 2022.

a) Assuming an exponential growth for the number of downloads, find the formula for the downloads depending on the year t.

b) Assuming the number of downloads will follow the formula from part (a), what will the number of downloads be in the year 2026?

c) In what year will the number of downloaded applications reach the 25 million barrier?

Exercise 15.5

The population size of a city was $79,000$ in the year 1998 and $136,000$ in the year 2013. Assume that the population size follows an exponential function.

a) Find the formula for the population size.

b) What is the population size in the year 2030?

c) What is the population size in the year 2035?

d) When will the city reach its expected maximum capacity of one million residents?

Exercise 15.6

The population of a city decreases at a rate of 2.3% per year. After how many years will the population be at 90% of its current size? Round your answer to the nearest tenth.

Exercise 15.7

A big company plans to expand its franchise and, with this, its number of employees. For tax reasons, it is most beneficial to expand the number of employees at a rate of 5% per year. If the company currently has 4730 employees, how many years will it take until the company has 6000 employees? Round your answer to the nearest hundredth.

Exercise 15.8

An ant colony has a population size of 4000 ants and is increasing at a rate of 3% per week. How long will it take until the ant population has doubled? Round your answer to the nearest tenth.

Exercise 15.9

The size of a beehive is decreasing at a rate of 15% per month. How long will it take for the beehive to be at half of its current size? Round your answer to the nearest hundredth.

Exercise 15.10

If the population size of the world is increasing at a rate of 0.5% per year, how long does it take until the world population doubles in size? Round your answer to the nearest tenth.