Mat 1375 HW14

Exercise 14.1

Combine the terms and write your answer as one logarithm.

a) $3\ln(x)+\ln(y)$

b) $\log(x)-\frac{2}{3}\log(y)$

c) $\frac{1}{3}\log(x)-\log(y)+4\log(z)$

d) $\log(xy^2z^3)-\log(x^4y^3z^2)$

e) $\frac{1}{4}\ln(x)-\frac{1}{2}\ln(y)+\frac{2}{3}\ln(z)$

f) $-\ln(x^2-1)+\ln(x-1)$

Sol: a) $3\ln(x)+\ln(y) \underset{\text{power rule}}{=} \ln(x^3)+\ln(y) \underset{\text{product rule}}{=} \ln(x^3y)$

b) $\log(x)-\frac{2}{3}\log(y) \underset{\text{power rule}}{=} \log(x)+\log(y^{-\frac{2}{3}}) \underset{\text{product rule}}{=} \log(x\cdot y^{-\frac{2}{3}})$

c) $\frac{1}{3}\log(x)-\log(y)+4\log(z) = \log(x^{\frac{1}{3}})+\log(y^{-1})+\log(z^4)$

$= \log(x^{\frac{1}{3}}\cdot y^{-1}\cdot z^4) = \log\left(\frac{x^{\frac{1}{3}}z^4}{y}\right)$

d) $\log(xy^2z^3)-\log(x^4y^3z^2) = \log\left(\frac{xy^2z^3}{x^4y^3z^2}\right) = \log\left(\frac{z}{x^3y}\right)$

e) $\frac{1}{4}\ln(x)-\frac{1}{2}\ln(y)+\frac{2}{3}\ln(z) = \ln(x^{\frac{1}{4}})+\ln(y^{-\frac{1}{2}})+\ln(z^{\frac{2}{3}})$

$= \ln(x^{\frac{1}{4}}\cdot y^{-\frac{1}{2}}\cdot z^{\frac{2}{3}})$

Exercise 14.2

Write the expressions in terms of elementary logarithms $u = \log_b(x)$, $v = \log_b(y)$, and $w = \log_b(z)$ (whichever are applicable). Assume that $x, y, z > 0$.

a) $\log(x^3 \cdot y)$ b) $\log(\sqrt[3]{x^2} \cdot \sqrt[4]{y^7})$ c) $\log\left(\sqrt{x \cdot \sqrt[3]{y}}\right)$

d) $\ln\left(\frac{x^3}{y^4}\right)$ e) $\ln\left(\frac{x^2}{\sqrt{y} \cdot z^2}\right)$ f) $\log_3\left(\sqrt{\frac{x \cdot y^3}{\sqrt{z}}}\right)$

Sol: Let $u = \log(x)$, $v = \log(y)$, $w = \log(z)$

a) $\log(x^3 \cdot y) = \log(x^3) + \log(y) = 3\log(x) + \log(y) = \boxed{3u + v}$

b) $\log(\sqrt[3]{x^2} \cdot \sqrt[4]{y^7}) = \log(x^{\frac{2}{3}} y^{\frac{7}{4}}) = \frac{2}{3}\log(x) + \frac{7}{4}\log(y) = \boxed{\frac{2}{3}u + \frac{7}{4}v}$

c) $\log(\sqrt{x \cdot \sqrt[3]{y}}) = \log(\sqrt{x} \cdot \sqrt{\sqrt[3]{y}}) = \log(x^{\frac{1}{2}}) + \log((y^{\frac{1}{3}})^{\frac{1}{2}})$

$= \frac{1}{2}\log(x) + \log(y^{\frac{1}{6}}) = \frac{1}{2}\log(x) + \frac{1}{6}\log(y) = \boxed{\frac{1}{2}u + \frac{1}{6}v}$

Let $u = \ln(x)$, $v = \ln(y)$, $w = \ln(z)$

d) $\ln\left(\frac{x^3}{y^4}\right) = \ln(x^3) - \ln(y^4) = 3\ln(x) - 4\ln(y) = 3u - 4v$

e) $\ln\left(\frac{x^2}{\sqrt{y} \cdot z^2}\right) = \ln(x^2) - \ln(\sqrt{y}) - \ln(z^2)$

$= 2\ln(x) - \frac{1}{2}\ln(y) - 2\ln(z) = 2u - \frac{1}{2}v - 2w$

Let $u = \log_3(x)$, $v = \log_3(y)$, $w = \log_3(z)$

f) $\log_3\left(\sqrt{\frac{xy^3}{\sqrt{z}}}\right) = \log_3\left(\frac{\sqrt{x} \cdot \sqrt{y^3}}{\sqrt{\sqrt{z}}}\right) = \log_3(\sqrt{x}) + \log_3(\sqrt{y^3}) - \log_3(\sqrt{\sqrt{z}})$

$= \log_3(x^{\frac{1}{2}}) + \log_3(y^{\frac{3}{2}}) - \log_3(z^{\frac{1}{4}})$

$= \frac{1}{2}\log_3(x) + \frac{3}{2}\log_3(y) - \frac{1}{4}\log_3(z) = \frac{1}{2}u + \frac{3}{2}v - \frac{1}{4}w$

Exercise 14.3

Solve for x without using a calculator.

a)	$\ln(2x+4) = \ln(5x-5)$	b)	$\ln(x+6) = \ln(x-2) + \ln(3)$
c)	$\log_2(x+5) = \log_2(x) + 5$	d)	$\log(x) + 1 = \log(5x+380)$
e)	$\log(x+5) + \log(x) = \log(6)$	f)	$\log_2(x) + \log_2(x-6) = 4$
g)	$\log_6(x) + \log_6(x-16) = 2$	h)	$\log_5(x-24) + \log_5(x) = 2$

Sol: (a) $\ln(2x+4) = \ln(5x-5)$

$\ln(x)$ is One-to-One $\Rightarrow 2x+4 = 5x-5$

$\Rightarrow 9 = 3x \Rightarrow \boxed{x=3}$

(since, when $x=3$, $2x+4>0$, $5x-5>0$, then $x=3$ is a solution)

(b) $\ln(x+6) = \ln(x-2) + \ln(3)$

product rule

$\ln(x+6) = \ln 3\cdot(x-2)$

one-to-one

$x+6 = 3(x-2)$

$\Rightarrow x+6 = 3x-6$

$\Rightarrow 12 = 2x \Rightarrow \boxed{x=6}$

(since, $x=6$, $x+6>0$, $x-2>0$, then $x=6$ is a solution)

e) $\log(x+5) + \log(x) = \log(6)$

Product rule

$\log x(x+5) = \log(6)$

one-to-one

$x(x+5) = 6$

$\Rightarrow x^2+5x-6=0 \Rightarrow (x-1)(x+6)=0$

$x \quad -1$

$x \quad +6$

$\Rightarrow \boxed{x=1}$ or $\cancel{-6}$

Check if $x+5>0$ & $x>0$

$x=1$ ✓

$x=-6$ ✗

f) $\log_2(x) + \log_2(x-6) = 4$

product rule

$\log_2 x(x-6) = 4$

switch to exponential form

$x(x-6) = 2^4$

$\Rightarrow x^2-6x-16=0$

$x \quad +2$

$x \quad -8$

$\Rightarrow (x+2)(x-8)=0 \Rightarrow x = \cancel{-2}$ or 8

Check if $x>0$ & $x-6>0$

$\Rightarrow x=-2$ ✗

$x=8$ ✓

g) $\log_6(x) + \log_6(x-16) = 2$

Product rule

$\log_6 x\cdot(x-16) = 2$

switch to exponential form

$x(x-16) = 6^2$

$x^2-16x-36=0$

$x \quad +2$

$x \quad -18$

$\Rightarrow (x+2)(x-18)=0$

$x=\cancel{-2}$ or $\boxed{x=18}$

Check if $x>0$ & $x-16>0$

$x=-2$ ✗

$x=18$ ✓

h) $\log_5 (x-24) + \log_5 (x) = 2$

product rule

$\log_5 x \cdot (x-24) = 2$

switch to exponential form

$x \cdot (x-24) = 5^2$

$x^2 - 24x - 25 = 0$

$x \quad +1$

$x \quad -25$

$\Rightarrow (x+1)(x-25) = 0$

$\Rightarrow x = \cancel{-1}$ or $\boxed{25}$

Check $x-24>0$ & $x>0$

$x=-1$ ✗

$x=25$ ✓