Mat 1375 HW13

Exercise 13.1

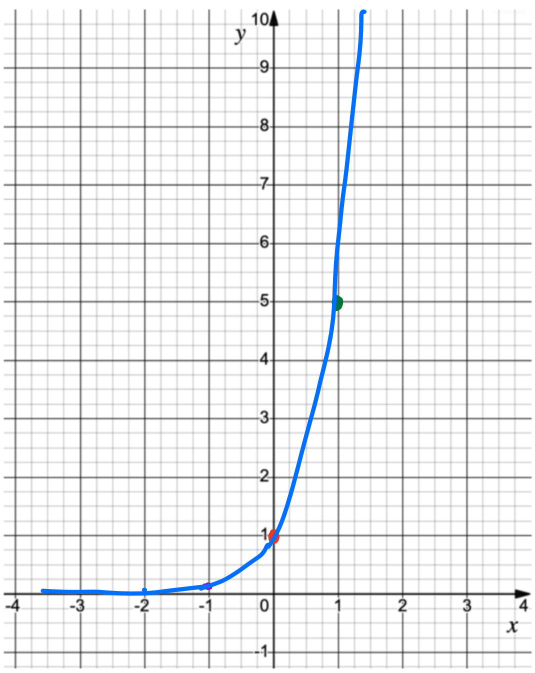
Graph the following functions with the calculator.

a) $y = 5^x$ b) $y = 1.01^x$ c) $y = (\frac{1}{3})^x$ d) $y = 0.97^x$

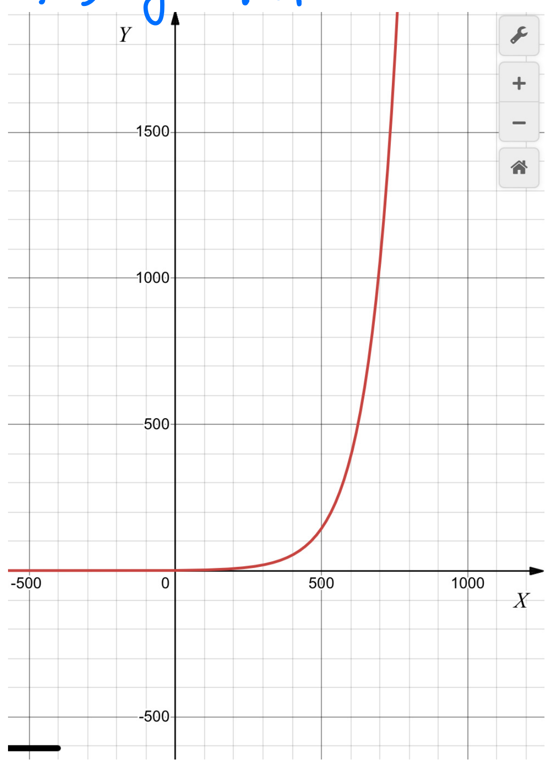
e) $y = 3^{-x}$ f) $y = (\frac{1}{3})^{-x}$ g) $y = e^{x^2}$ h) $y = 0.01^x$

Sol: a) $f(x) = y = 5^x$

x	$f(x)$
-3	$\frac{1}{125}$
-2	$\frac{1}{25}$
-1	$\frac{1}{5}$
0	1
1	5
2	25
3	125

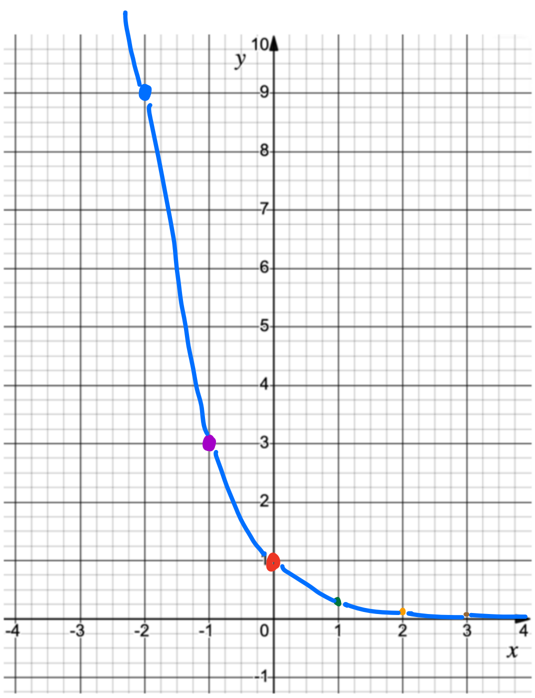


b) $f(x) = y = 1.01^x$

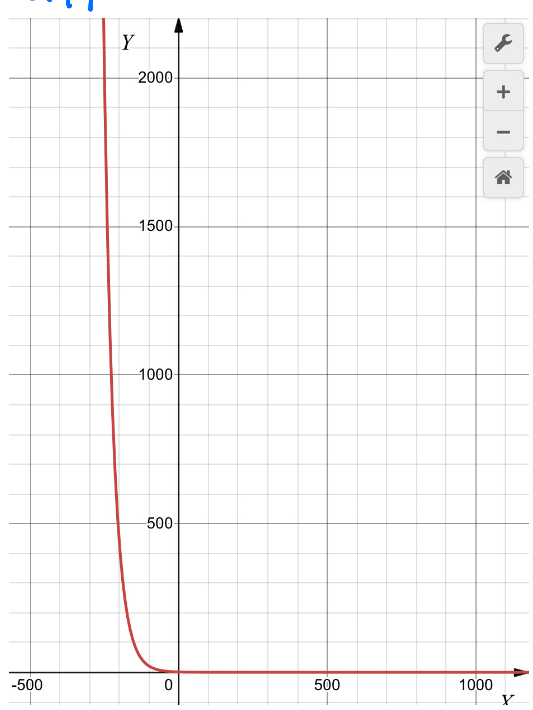


c) $f(x) = y = (\frac{1}{3})^x$

x	$f(x)$
-3	27
-2	9
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$
3	$\frac{1}{27}$

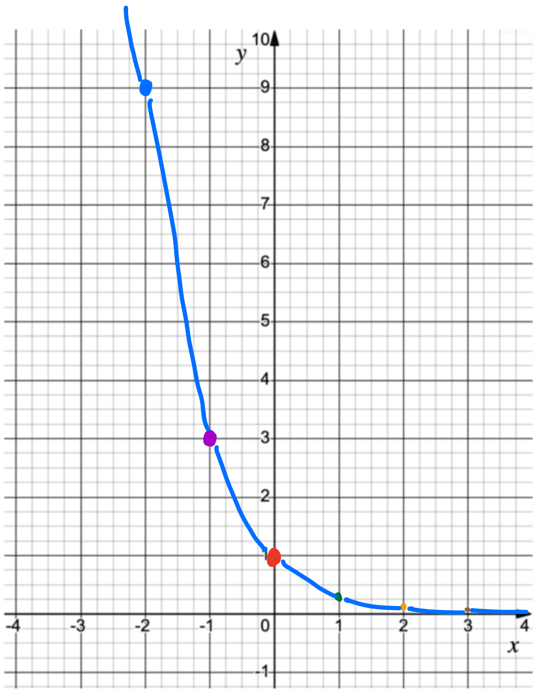


d) $f(x) = y = 0.97^x$



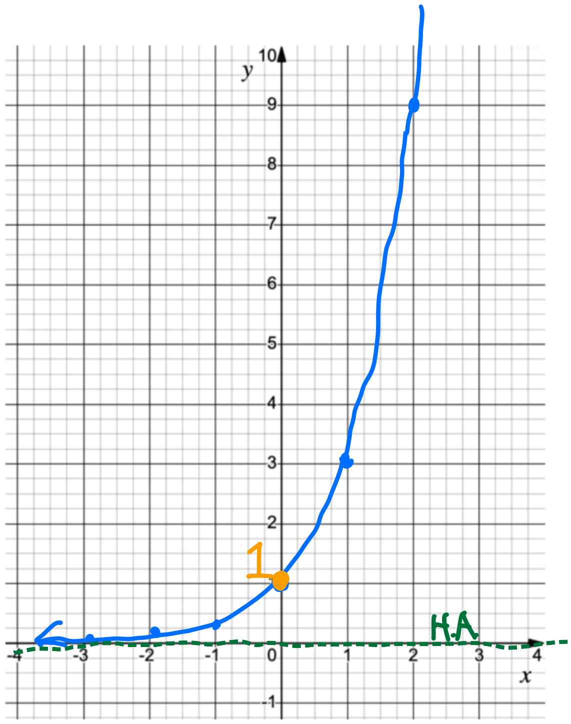
e) $f(x) = y = 3^{-x}$

x	$f(x)$
-3	27
-2	9
-1	3
0	1
1	$\frac{1}{3}$
2	$\frac{1}{9}$
3	$\frac{1}{27}$



f) $f(x) = y = \left(\frac{1}{3}\right)^{-x}$

x	$f(x)$
-3	$\frac{1}{27}$
-2	$\frac{1}{9}$
-1	$\frac{1}{3}$
0	1
1	3
2	9
3	27

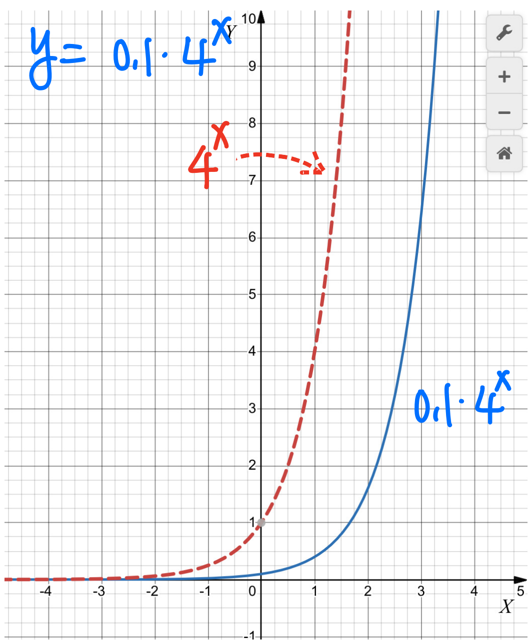


Exercise 13.2

Graph the given function. Describe how the graph is obtained by a transformation from the graph of an exponential function $y = b^x$ (for appropriate base b).

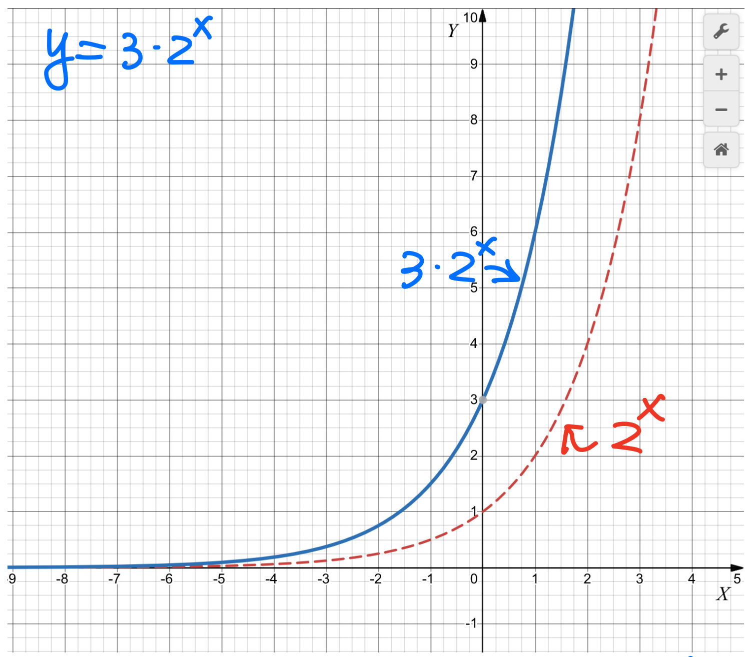
✓ a) $y = 0.1 \cdot 4^x$ ✓ b) $y = 3 \cdot 2^x$ ✓ c) $y = (-1) \cdot 2^x$
✓ d) $y = 0.006 \cdot 2^x$ ✓ e) $y = e^{-x}$ f) $y = e^{-x} + 1$

Sol. a) $y = 0.1 \cdot 4^x$



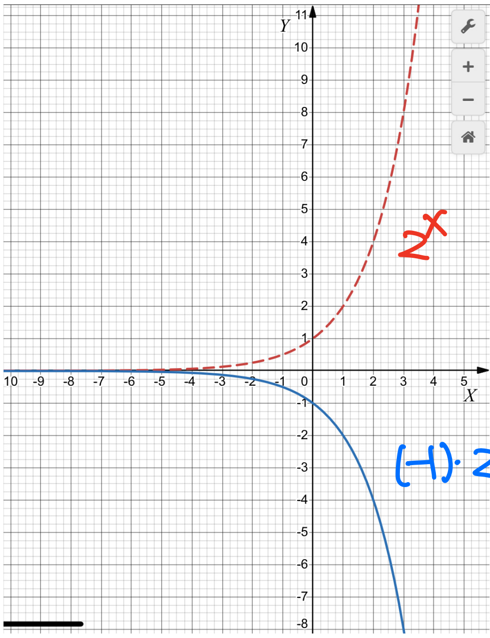
For the same x, the value $0.1 \cdot 4^x$ is $\frac{1}{10}$ of 4^x

b) $y = 3 \cdot 2^x$



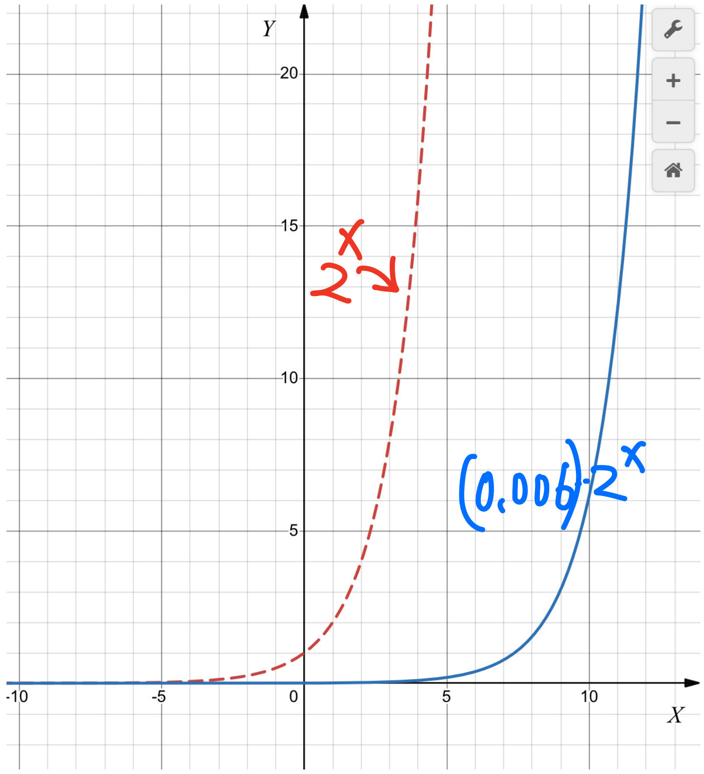
For the same x, the value of $3 \cdot 2^x$ is 3 times of 2^x.

c)

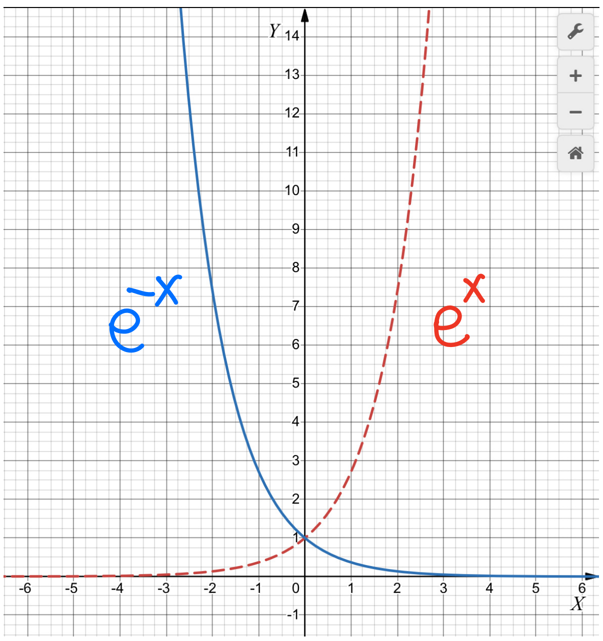


$(-1)\cdot 2^x$ is the graph 2^x reflecting about x-axis

d)



e)



e^{-x} is the graph e^x reflecting about y-axis.

Exercise 13.4

Evaluate the following expressions *without* using a calculator.

a) $\log_7(49)$	b) $\log_3(81)$	c) $\log_2(64)$	d) $\log_{50}(2500)$
e) $\log_2(0.25)$	f) $\log(1000)$	g) $\ln(e^4)$	h) $\log_{13}(13)$
i) $\log(0.1)$	j) $\log_6(\frac{1}{36})$	k) $\ln(1)$	l) $\log_{\frac{1}{2}}(8)$

Sol: a) $\log_7(49) = \log_7(7^2) = 2\cdot\log_7(7) = 2\cdot 1 = 2$

b) $\log_3(81) = \log_3(3^4) = 4\cdot\log_3(3) = 4\cdot 1 = 4$

c) $\log_2(64) = \log_2(2^6) = 6 \cdot \log_2(2) = 6 \cdot 1 = 6$

d) $\log_{50}(2500) = \log_{50}(50^2) = 2 \cdot \log_{50}(50) = 2 \cdot 1 = 2$

e) $\log_2(0.25) = \log_2(\frac{1}{4}) = \log_2(2^{-2}) = -2 \cdot \log_2(2) = -2 \cdot 1 = -2.$

f) $\log(1000) = \log_{10}(10^3) = 3 \cdot \log_{10}(10) = 3 \cdot 1 = 3$

g) $\ln(e^4) = 4 \cdot \ln(e) = 4 \cdot 1 = 4$

h) $\log_{13}(13) = 1$

i) $\log(0.1) = \log_{10}(10^{-1}) = -1 \cdot \log_{10}(10) = -1 \cdot 1 = -1$

j) $\log_6(\frac{1}{36}) = \log_6(6^{-2}) = -2 \cdot \log_6(6) = -2 \cdot 1 = -2$

k) $\ln(1) = \ln(e^0) = 0 \cdot \ln(e) = 0 \cdot 1 = 0.$

l) $\log_{\frac{1}{2}}(8) = \log_{\frac{1}{2}}(\frac{1}{2}^{(-3)}) = -3 \cdot \log_{\frac{1}{2}}(\frac{1}{2}) = -3 \cdot 1 = -3$

$8 = 2^3 = (2^{-1})^{(-3)} = (\frac{1}{2})^{-3}$

Exercise 13.5

Using a calculator, approximate the following expressions to the nearest thousandth.

a) $\log_3(50)$ $= 3.560$

b) $\log_3(12)$ $= 2.261$

c) $\log_{17}(0.44)$ $= -0.289$

d) $\log_{0.34}(200)$ $= -4.911$

Exercise 13.6

State the domain of the function f and find any vertical asymptote(s) and x-intercept(s). Use the results to sketch the graph.

a) $f(x) = \log(x)$ b) $f(x) = \log(x+7)$
c) $f(x) = \ln(x+5) - 1$ d) $f(x) = \ln(3x-6)$
e) $f(x) = 2 \cdot \log(x+4)$ f) $f(x) = -4 \cdot \log(x+2)$
g) $f(x) = \log_3(7x+5)$ h) $f(x) = \ln(-6x+14)$

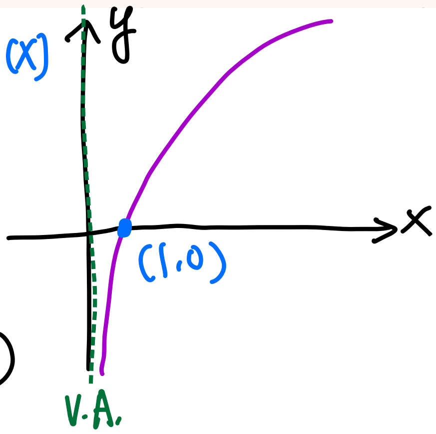
Sol: a) $f(x) = \log(x)$

Domain: $(0, \infty)$

V.A.: $x = 0$

x-intercept: $(1, 0)$

(when $f(x) = 0 \Rightarrow x = 1$)



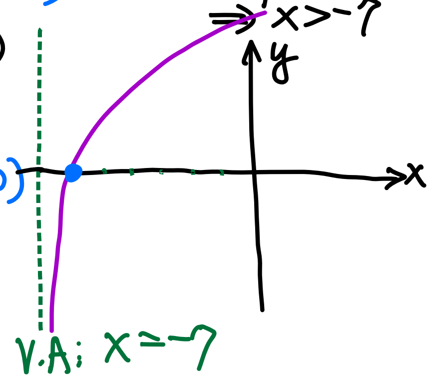
b) $f(x) = \log(x+7) \Rightarrow x+7 > 0$
$\Rightarrow x > -7$

Domain: $(-7, \infty)$

V.A. $x = -7$

x-intercept: $(-6, 0)$

(when $f(x) = 0$
$\Rightarrow x+7 = 1$
$\Rightarrow x = -6$)



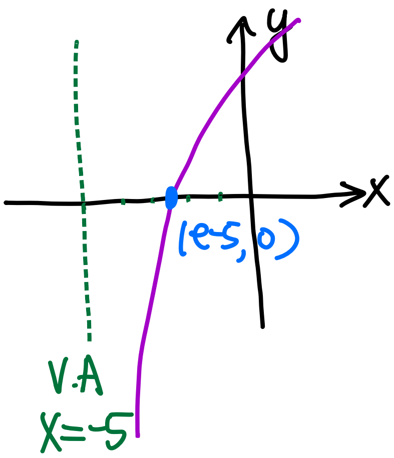
c) $f(x) = \ln(x+5) - 1 \Rightarrow x+5 > 0 \Rightarrow x > -5$

Domain: $(-5, \infty)$

V.A. $x = -5$

x-intercept $(e-5, 0)$

(when $f(x) = 0 \Rightarrow$
$\ln(x+5) - 1 = 0$
$\Rightarrow \ln(x+5) = 1$
$\Rightarrow x+5 = e^1$
$\Rightarrow x = e^1 - 5$



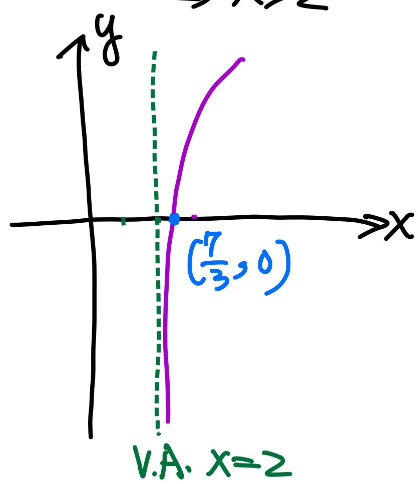
d) $f(x) = \ln(3x-6) \Rightarrow 3x-6 > 0 \Rightarrow 3x > 6$
$\Rightarrow x > 2$

Domain: $(2, \infty)$

V.A. $x = 2$

x-intercept $(\frac{7}{3}, 0)$

(when $f(x) = 0$
$\Rightarrow \ln(3x-6) = 0$
$\Rightarrow 3x-6 = 1$
$\Rightarrow 3x = 7$
$\Rightarrow x = \frac{7}{3}$)



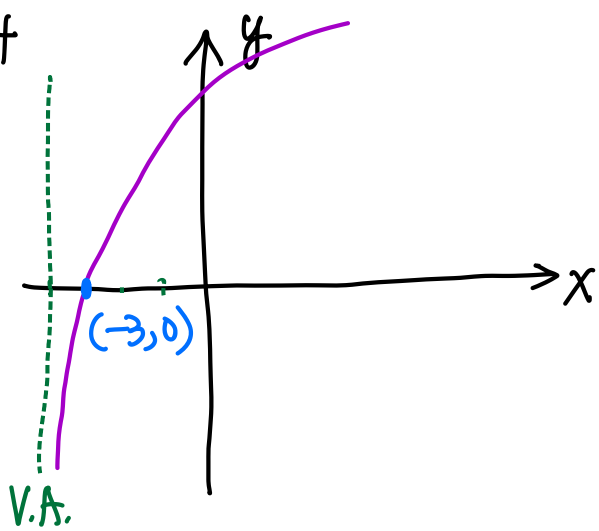
e) $f(x) = 2 \cdot \log(x+4) \Rightarrow x+4 > 0 \Rightarrow x > -4$

Domain: $(-4, \infty)$

V.A. $x = -4$

x-intercept $(-3, 0)$

(when $f(x) = 0 \Rightarrow 2 \cdot \log(x+4) = 0$
$\Rightarrow \log(x+4) = 0$
$\Rightarrow x+4 = 10^0 = 1$
$\Rightarrow x = -3$)



f) $f(x) = -4 \cdot \log(x+2) \Rightarrow x+2>0 \Rightarrow x>-2$

Domain: $(-2, \infty)$

V.A. $x=-2$

X-intercept $(-1, 0)$

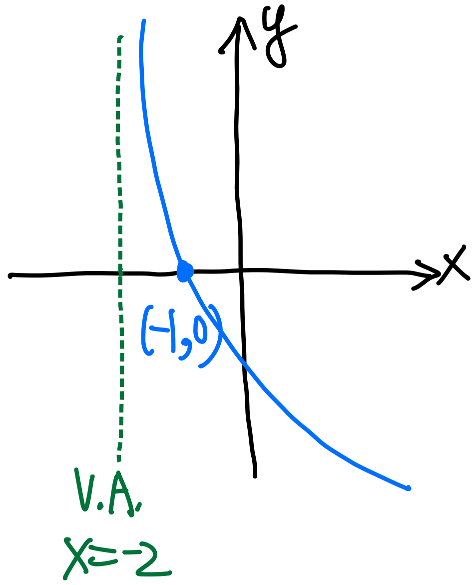
(when $f(x)=0$,

$\Rightarrow -4 \cdot \log(x+2) = 0$

$\Rightarrow \log(x+2) = 0$

$\Rightarrow x+2 = 10^0 = 1$

$\Rightarrow x = -1$)



g) $f(x) = \log_3(7x+5) \Rightarrow 7x+5>0$

$\Rightarrow 7x > -5$

$\Rightarrow x > -\frac{5}{7}$

Domain $(-\frac{5}{7}, \infty)$

V.A. $x = -\frac{5}{7}$

X-intercept $(-\frac{4}{7}, 0)$

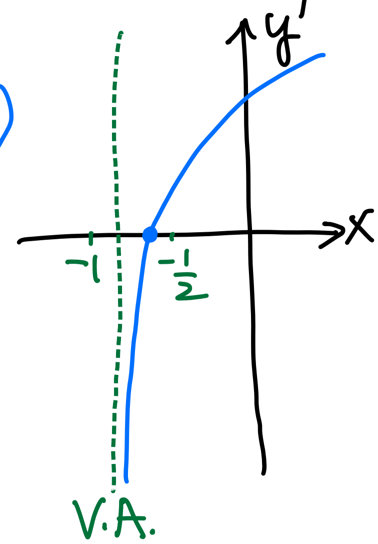
(when $f(x)=0$,

$\Rightarrow \log_3(7x+5) = 0$

$\Rightarrow 7x+5 = 3^0 = 1$

$\Rightarrow 7x = -4$

$\Rightarrow x = -\frac{4}{7}$



h) $f(x) = \ln(-6x+14) \Rightarrow -6x+14>0$

$\Rightarrow 6x < 14$

$\Rightarrow x < \frac{14}{6} = \frac{7}{3}$

Domain: $(-\infty, \frac{7}{3})$

V.A. $x = \frac{7}{3}$

X-intercept $(\frac{13}{6}, 0)$

($f(x)=0 \Rightarrow \ln(-6x+14)=0$

$\Rightarrow -6x+14 = e^0 = 1$

$\Rightarrow -6x = -13$

$\Rightarrow x = \frac{13}{6}$