Mat 1375 HW 12

Exercise 12.1

Solve for x.

a) $5x + 6 \leq 21$

b) $3 + 4x > 10x$

Sol a) $5x + 6 \leq 21$

$\Rightarrow 5x \leq 21 - 6$

$\Rightarrow \frac{5x}{5} \leq \frac{15}{5}$

$\Rightarrow \boxed{x \leq 3}$

b) $3 + 4x > 10x$

$\Rightarrow 3 > 10x - 4x$

$\Rightarrow \frac{3}{6} > \frac{6x}{6}$

$\Rightarrow \boxed{\frac{1}{2} > x}$

Exercise 12.2

Solve for x.

a) $x^2 - 5x - 14 > 0$

b) $x^2 - 2x \geq 35$

c) $x^2 - 4 \leq 0$

d) $x^2 + 3x - 3 < 0$

e) $2x^2 + 2x \leq 12$

f) $3x^2 < 2x + 1$

g) $x^2 - 4x + 4 > 0$

h) $x^3 - 2x^2 - 5x + 6 \geq 0$

Sol: a) $x^2 - 5x - 14 > 0$

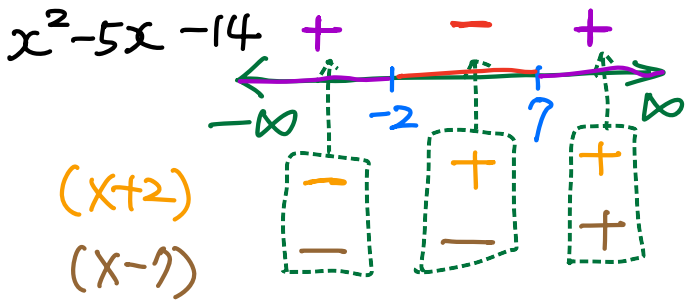
① Find root(s) of $x^2 - 5x - 14 = 0 \Rightarrow (x+2)(x-7) = 0$

$x \quad +2$

$x \quad -7$

$\Rightarrow x + 2 = 0$ or $x - 7 = 0 \Rightarrow x = -2, x = 7.$

② Number line for positivity/negativity analysis:



For $x^2 - 5x - 14 > 0$, we have

$x \in (-\infty, -2) \cup (7, \infty)$

③ Check endpoint: since the question "$x^2 - 5x - 14 > 0$" is without "equal", then endpoints are NOT included.

$\Rightarrow x \in (-\infty, -2) \cup (7, \infty)$

b) $x^2-2x \geq 35$

$\Rightarrow x^2-2x-35 \geq 0$

① Find the root of $x^2-2x-35=0 \Rightarrow (x+5)(x-7)=0$

$x \quad +5$

$x \quad -7$

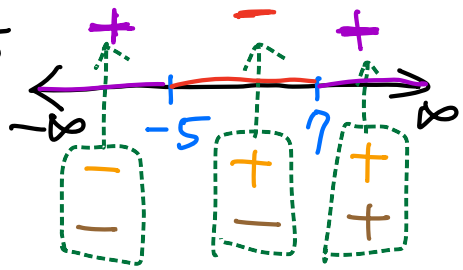
$\Rightarrow x+5=0$ or $x-7=0 \Rightarrow x=-5$ or $x=7$

② Number line for positivity/negativity analysis:

$x^2-2x-35$

$(x+5)$

$(x-7)$



For $x^2-2x-35>0$, we have

$x \in (-\infty, -5) \cup (7, \infty)$

③ Check end points: Since the question "$x^2-2x-35 \geq 0$" is with equal. then endpoints are included!

$\Rightarrow x \in (-\infty, -5] \cup [7, \infty)$

---

c) $x^2-4 \leq 0$

① Find the roots of $x^2-4=0 \Rightarrow (x+2)(x-2)=0$

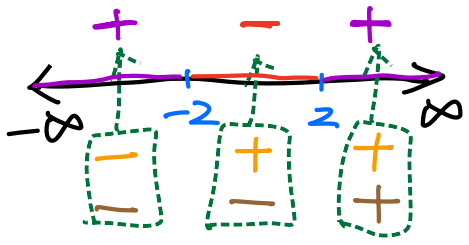
$\Rightarrow x=2$ or $x=-2$.

② Number line for positivity/negativity analysis:

x^2-4

$(x+2)$

$(x-2)$



For $x^2-4<0$, we have

$x \in (-2, 2)$

③ Check end points: Since the question "$x^2-4 \leq 0$" is with equal then endpoints are included!

$\Rightarrow x \in [-2, 2]$

---

h) $x^3-2x^2-5x+6 \geq 0$

① Find the roots of $x^3-2x^2-5x+6=0$.

- Test educated guess "$x=1$":

$(1)^3-2\cdot(1)^2-5\cdot(1)+6=1-2-5+6=0$

$\Rightarrow$ $x=1$ is a root and $(x-1)$ is a factor of x^3-2x^2-5x+6

- Then find all roots by factor x^3-2x^2-5x+6.

$x^3-2x^2-5x+6=(x-1)\cdot(x^2-x-6)=0$

$x\cdot x=x^2$, $+2-3=-x$

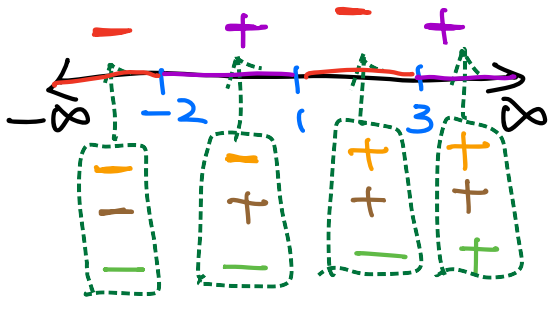
	1	−2	−5	+6
1		1	−1	−6
	1	−1	−6	0

$\Rightarrow (x-1)\cdot(x+2)\cdot(x-3)=0$

$\Rightarrow x=1$ or $x=-2$ or $x=3$.

② Number line for positivity/negativity analysis:

x^3-2x^2-5x+6



$(x-1)$
$(x+2)$
$(x-3)$

For $x^3-2x^2-5x+6>0$, we have

$x\in(-2,1)\cup(3,\infty)$

③ Check end points: Since the question "$x^3-2x^2-5x+6\geq 0$" is with "equal", then endpoints are included!

$\Rightarrow x\in[-2,1]\cup[3,\infty)$

Exercise 12.4

Solve for x.

a) $|2x+7|>9$ ✓ b) $|6x+2|<3$ ✓

c) $|5-3x|\geq 4$ ✓ d) $|-x-7|\leq 5$

Sol: a) $|2x+7|>9$

① Solve x for $|2x+7|=9$ $\Rightarrow 2x+7=9$ or $2x+7=-9$

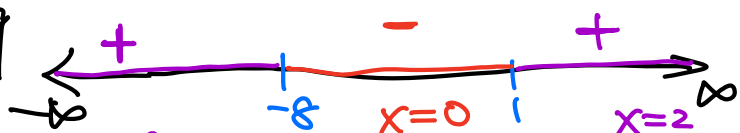
$\Rightarrow 2x=2$ or $2x=-16$ $\Rightarrow x=1$ or $x=-8$

② Number line

For $|2x+7|-9>0$

$x\in(-\infty,-8)\cup(1,\infty)$

$|2x+7|-9$



$x=-9$: $|2\cdot(-9)+7|-9=|-18+7|-9=11-9>0$

$x=0$: $|2\cdot 0+7|-9=7-9<0$

$x=2$: $|2\cdot 2+7|-9=11-9>0$

③ Check endpoints: Since the question "$|2x+7|-9>0$" is without equal, then endpoints are NOT included.

$\Rightarrow x \in (-\infty, -8) \cup (1, \infty)$

---

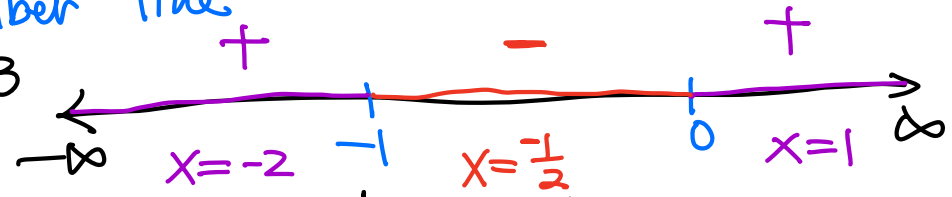
b) $|6x+2|<3$

① Solve $|6x+3|=3 \Rightarrow 6x+3=3$ or $6x+3=-3$

$\Rightarrow 6x=0$ or $6x=-6 \Rightarrow x=0$ or $x=-1$.

② Number line

$|6x+2|-3$



$|6(-2)+2|-3 = |-10|-3 = 10-3=7>0$

$|6(-\frac{1}{2})+2|-3 = |-3+2|-3 = 1-3<0$

$|6\cdot(1)+2|-3 = |8|-3 = 8-3=5>0$

For $|6x+2|-3<0 \Rightarrow x \in (-1, 0)$

③ Check endpoints: Since the question "$|6x+2|-3<0$" is without equal, then endpoints are NOT included.

$\Rightarrow x \in (-1, 0)$

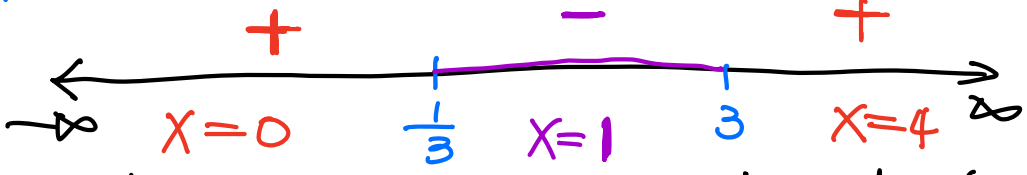
---

c) $|5-3x| \geq 4$

① Solve x for $|5-3x|=4 \Rightarrow 5-3x=4$ or $5-3x=-4$

$\Rightarrow -3x=-1$ or $-3x=-9 \Rightarrow x=\frac{1}{3}$ or $x=3$

② Number line



$|5-3\cdot0|-4 = 5-4=1>0$

$|5-3\cdot1|-4 = |2|-4 = 2-4=-2<0$

$|5-3\cdot4|-4 = |5-12|-4 = 7-4>0$

For $|5-3x|>4$, we have $x \in (-\infty, \frac{1}{3}) \cup (3, \infty)$

③ Check endpoints: since the question "$|5-3x| \geq 4$" is with equal, then endpoints are included!

$x \in (-\infty, \frac{1}{3}] \cup [3, \infty)$

Exercise 12.5

Solve for x.

a) $\frac{x+2}{x+4} \geq 0$ b) $\frac{x-5}{2-x} > 0$ c) $\frac{9x-11}{7x+15} \leq 0$ d) $\frac{13x+4}{6x-1} \geq 0$

e) $\frac{7x-2}{3x+8} < 0$ f) $\frac{4x-4}{x^2-4} \geq 0$ g) $\frac{x-2}{x^2-4x-5} < 0$ h) $\frac{x^2-9}{x^2-4} \geq 0$

Sol: a) $\frac{x+2}{x+4} \geq 0$

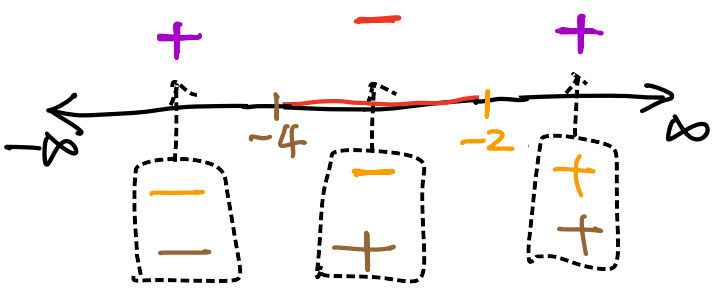
① Find the x-interopt(s) and vertical asymptote(s).

$x+2=0 \Rightarrow x=-2$

$x+4=0 \Rightarrow x=-4$

② Number line

$\frac{x+2}{x+4}$



$x+2$

$x+4$

For $\frac{x+2}{x+4} > 0$

$x \in (-\infty, -4) \cup (-2, \infty)$

③ Check endpoints:

$x=-2 \Rightarrow \frac{-2+2}{-2+4}=0$ ✓ $\Rightarrow x \in (-\infty, -4) \cup [-2, \infty)$

$x=-4 \Rightarrow \frac{x+2}{x+4}$ undefined

c) $\frac{9x-11}{7x+15} \leq 0$

① Find x-intercepts and a vertical asymptote

$9x-11=0$

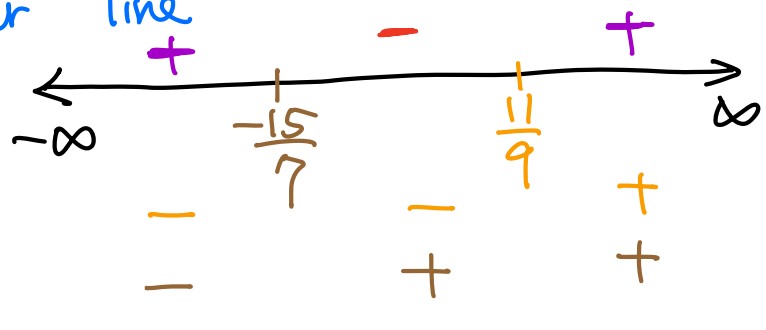
$x=\frac{11}{9}$

$7x+15=0$

$x=-\frac{15}{7}$

② Number line

$\frac{9x-11}{7x+15}$



$9x-11$: $-$ | $-$ | $+$

$7x+15$: $-$ | $+$ | $+$

For $\frac{9x-11}{7x+15} < 0$, we have

$x \in (-\frac{15}{7}, \frac{11}{9})$

③ Check endpoints

$x = \frac{11}{9}$, $\frac{9\cdot\frac{11}{9}-11}{7\cdot\frac{11}{9}+5} = 0$ ✓ $\Rightarrow x \in (-\frac{15}{7}, \frac{11}{9}]$

$x = -\frac{15}{7}$ $\frac{9x-11}{7x+15}$ undefined ✗

d) $\frac{13x+4}{6x-1} \geq 0$

① Find x-intercept and the vertical asymptote

$13x+4=0$

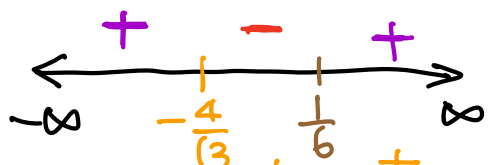
$x = -\frac{4}{13}$

$6x-1=0$

$x = \frac{1}{6}$

② Number line

$\frac{13x+4}{6x-1}$



$(13x+4)$: $-$ | $+$ | $+$

$(6x-1)$: $-$ | $-$ | $+$

For $\frac{13x+4}{6x-1} > 0$, we have

$x \in (-\infty, -\frac{4}{13}) \cup (\frac{1}{6}, \infty)$

③ Check the endpoints

$x = -\frac{4}{13}$ $\frac{13(-\frac{4}{13})+4}{6(-\frac{4}{13})+1} = 0$ ✓ $\Rightarrow x \in (-\infty, -\frac{4}{13}] \cup (\frac{1}{6}, \infty)$

$x = \frac{1}{6}$ made $\frac{13x+4}{6x+1}$ undefined ✗

g) $\frac{x-2}{x^2-4x-5} < 0 \Rightarrow \frac{x-2}{(x-5)(x+1)} < 0$

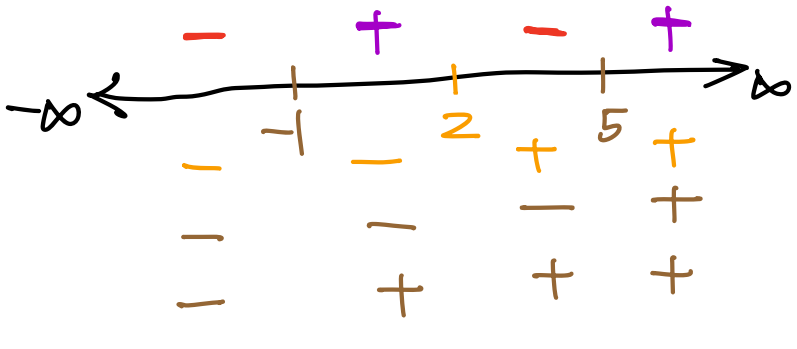
① Find x-intercept and vertical asymptotes

$x-2=0 \Rightarrow x=2$

$x-5=0$ and $x+1=0 \Rightarrow x=5, \quad x=-1$

② Number line

$\frac{x-2}{(x+1)(x-5)}$



$(x-2)$	−	−	+	+
$(x-5)$	−	−	−	+
$(x+1)$	−	+	+	+

For $\frac{x-2}{x^2-4x-5} < 0$, we have

$x \in (-\infty, -1) \cup (2, 5)$

③ Check endpoints: there is no need to check endpoints because the question "$\frac{x-2}{x^2-4x-5} < 0$" has no equal sign.

$\Rightarrow x \in (-\infty, -1) \cup (2, 5)$