

### 10.3 Exercises

#### ✓ Exercise 10.1

Find the domain, the vertical asymptotes, and removable discontinuities of the functions:

$$\begin{array}{ll} \text{a) } f(x) = \frac{2}{x-2} & \text{b) } f(x) = \frac{x^2+2}{x^2-6x+8} \\ \text{c) } f(x) = \frac{3x+6}{x^3-4x} & \text{d) } f(x) = \frac{(x-2)(x+3)(x+4)}{(x-2)^2(x+3)(x-5)} \\ \text{e) } f(x) = \frac{x-1}{x^3-1} & \text{f) } f(x) = \frac{2}{x^3-2x^2-x+2} \end{array}$$

#### ✓ Exercise 10.2

Find the horizontal asymptotes of the functions:

$$\begin{array}{ll} \text{a) } f(x) = \frac{8x^2+2x+1}{2x^2+3x-2} & \text{b) } f(x) = \frac{1}{(x-3)^2} \\ \text{c) } f(x) = \frac{x^2+3x+2}{x-1} & \text{d) } f(x) = \frac{12x^3-4x+2}{-3x^3+2x^2+1} \end{array}$$

#### ✓ Exercise 10.3

Find the  $x$ - and  $y$ -intercepts of the functions:

$$\begin{array}{ll} \text{a) } f(x) = \frac{x-3}{x-1} & \text{b) } f(x) = \frac{x^3-4x}{x^2-8x+15} \\ \text{c) } f(x) = \frac{(x-3)(x-1)(x+4)}{(x-2)(x-5)} & \text{d) } f(x) = \frac{x^2+5x+6}{x^2+2x} \end{array}$$

#### Exercise 10.4

Sketch a complete graph of the function  $f$ . To this end, calculate the domain of  $f$ , the horizontal and vertical asymptotes, the removable singularities, the  $x$ - and  $y$ -intercepts of the function, and graph the function with the graphing calculator.

$$\begin{array}{ll} \text{✓ a) } f(x) = \frac{7x+2}{3x-5} & \text{✓ b) } f(x) = \frac{x^2-x-2}{x^2+2x-3} \\ \text{c) } f(x) = \frac{3x^2-7x+2}{x^2-3x-10} & \text{d) } f(x) = \frac{x^2+7x+12}{x^2+6x+8} \\ \text{e) } f(x) = \frac{x-3}{x^3-3x^2-6x+8} & \text{f) } f(x) = \frac{x^3-3x^2-x+3}{x^3-2x^2} \end{array}$$

**Exercise 10.5**

Find a rational function  $f$  that satisfies all the given properties.

- a) vertical asymptote at  $x = 4$  and horizontal asymptote at  $y = 0$
- b) vertical asymptotes at  $x = 2$  and  $x = 3$  and horizontal asymptote at  $y = 5$
- c) removable singularity at  $x = 1$  and no horizontal asymptote